Economía
&SOCIEDAD

ECONOMIC GROWTH: PRODUCTIVITY, THRIFT AND CAPITAL ACCUMULATION

CRECIMIENTO ECONÓMICO: PRODUCTIVIDAD, AHORRO Y ACUMULACIÓN DE CAPITAL

CRESCIMENTO ECONÓMICO: PRODUTIVIDADE, POUPANÇA E ACUMULAÇÃO DE CAPITAL

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Abstract

The aim of this research is to study the relationship between thrifts, capital accumulation, and economic growth, as suggested by Swan in 1956. It provides a model which measures resource contribution, productivity, and share on economic growth and explains how it could occur in an economy. New formulas emerge as powerful tools in building a rigorous *new method of attack* to support the analysis and explanation of this subject, which was evaluated by the case of Costa Rica during the period 2010 – 2017. The theoretical and empirical findings of the analysis of capital and labor productivity, contribution, and share on economic growth support Swan's suggestion.

Key Words: Trevor Swan; Resource composition; Convergence; Depreciation; *Residue*; Costa Rica

Resumen

El objetivo de esta investigación es estudiar la relación entre el nivel de ahorro, la acumulación de capital y el crecimiento económico, propuesta por Swan en 1956. Provee un modelo que mide la contribución, productividad y participación de los recursos en el crecimiento económico y explicar cómo podría ocurrir en una economía. Nuevas fórmulas surgen

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como poderosas herramientas en la construcción de un riguroso *método de ataque* y apoyan el análisis y las explicaciones sobre ese tema, con el cual se evalúa el caso de Costa Rica en (2010 – 2017). Los resultados teóricos y empíricos soportan la sugerencia de Swan al analizar la productividad, contribución y participación del capital y el trabajo en el crecimiento económico.

Palabras claves: Trevor Swan; composición de los recursos; Productividad; Convergencia; Depreciación; *Residuo*; Costa Rica

Resumo

O objetivo desta pesquisa é estudar a relação entre o nível de poupança, acumulação de capital e crescimento econômico, proposto por Swan em 1956. Ele fornece um modelo que mede a contribuição, a produtividade e a participação de recursos no crescimento econômico e explica como isso poderia acontecer em uma economia. Novas fórmulas emergem como ferramentas poderosas na construção de um método rigoroso de ataque e apóiam as análises e explicações sobre esse tópico, com as quais o caso da Costa Rica é avaliado em (2010 - 2017). Os resultados teóricos e empíricos corroboram a sugestão de Swan ao analisar a produtividade, contribuição e participação do capital e do trabalho no crescimento econômico.

Palavras-chaves: Trevor Swan; Composição de recursos; Produtividade; Convergência; Depreciação; Resíduos; Costa Rica

Introduction

Trevor Swan's ([1956](#)) contribution to the theory of economic growth has not received the attention it deserves. He theoretically worked on economic growth and capital accumulation "to illustrate…a theme common to Adam Smith, Mill, and Lewis, the theory of which is perhaps best seen in Ricardo: namely, the connexion between capital accumulation and the growth of the productive labour force," and he also affirmed that "the neoclassical economists were in favour of productivity and thrift, but never found a way to make much use of them" (Swan, [1956](#), p. 334).

Based on Cobb-Douglas's ([1928](#)) production function, Swan's *unclassical* case attributes production growth to the combination of capital and labor force, and it professes to be an explanation of how saving and capital accumulation are related to production growth. It can be suggested that Swan tried to answer the five questions formulated by Cobb and Douglas ([1928](#), p. 139-140) in "A Theory of Production," adding the theme of thrift in relation to adding capital

to stock in a given instant. The role of savings in the capital accumulation process was also a concern in Keynes (1936), Harrod (1939; 1960), Domar (1946; 1953), Solow (1956), and more recently in Piketty (2014) and Stiglitz (2015).

Using the Keynesian concept of effective demand, Swan (1956) formulated two basic assumptions. First, "effective demand is so regulated (via the rate of interest or otherwise) that all savings are profitably invested, productive capacity is fully utilized, and the level of employment can never be increased merely by raising the level of spending" (p. 335). In Keynes (1936), the substance of his theory is effective demand, which is summed up into eight propositions (p. 22-23); it is defined as "the expected proceeds" at each instant, where the level of employment is not predicted to be *equal* to full employment (p. 22). Secondly, "the forces of perfect competition drive the rate of profit or interest r and the (real) wage rate w into equality with the marginal productivities of capital and labour" (Swan, 1956, p. 335).

Swan's arguments and crucial assumptions are useful in the present research to demonstrate how the rate of thrift and the rate of capital accumulation are related to the rate of labor and capital, and also capital and labor contribution to production growth. Swan's model hopes to reveal a strong relationship between capital and labor in production processes through their contributions to production growth. A result of this research is providing formulas to compute relative, average, and gross product, aggregated by those resources to production growth and both capital return and labor income or wage could be determined; by applying the model we propose to Costa Rican data during 2010 – 2017, all these indicators were obtained.

The next section analyzes the model suggested by Swan (1956) and as a result, new formulas will be provided, which measure the elasticity of resource composition, capital and labor productivity, and contribution and share on production growth. Section three focuses on expanding Swan's *unclassical* model, the fundamental challenge of this research, to reveal how Swan's assumptions can be explained. In the subsequent section, formulas will be provided to measure resource share on production growth while the following section will deal with the relationship between thrifts, accumulation, and economic growth. As an addendum, section six suggests a hybrid Swan-Solow equation of economic growth. To evaluate the robustness of the model we propose in this research, we analysis the case of Costa Rica during 2010 – 2017 in section seven. Furthermore, conclusions, limitations, and possible contributions of this research are stated at the end, followed by the references.

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Swan's *Unclassical* Case: Resource Contribution

"In the first instance…in a given state of the arts, the annual output Y depends on the stock of capital K and the labour force N" (Swan, 1956, p. 334).[2] Swan's case is based on Cobb-Douglas's (1928) production function. Therefore:

(1) $Y = F(K^{\alpha}L^{\beta}); \{(\alpha + \beta) = 1;\ \forall(\alpha, \beta)\mathbb{R}: (0,1)\}$.

Also, "the annual addition to the capital stock is the amount saved (sY), where (s) is a given ratio of saving to output (or income)" (Swan, 1956, p. 335).

(2) $s = \dfrac{S}{Y}$

Thus, from the previous equation, the amount saved is:

(3) $S = sY$

This amount saved is equal to *the annual addition to the capital stock*.

Now, *the annual addition to the capital stock* is the amount in which capital stock changes. Let us call Ḱ that addition and define it as follows:

(4) $Ḱ = sY$

Here we derived $(Ḱ = S)$ by equalizing equations (3) and (4). So, *the annual relative growth of capital* (k) is:

(5) $k = \dfrac{Ḱ}{K}$

So that:

(6) $Ḱ = kK$

After replacing equation (4), we obtain:

(7) $sY = kK$

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[2] Even Swan's phrase "in a given state of the arts" is similar to "in a given situation of technique" (Keynes, 1936, p. 20).

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Thus:

$$(8)\ k = s\frac{Y}{K}$$

Differentiating equation (1) presumes "all variables are treated as continuous functions of time, which is measured in years…'annual output' is the instantaneous rate of output per annum" (Swan, 1956, p. 335). This is as a consequence of annual variations in (K, L) and the elasticity of production (α, β) due to those changes: $dY = dF(K^\alpha L^\beta)$. From this result, the "instantaneous relative rates of growth per annum, subject to instantaneous compounding" (Swan, 1956, p. 335) (y) will emerge:[3]

$$(9)\ y = \alpha k + \beta n$$

where (n) "stand[s] for the annual relative rate of growth" of labor (Swan, 1956, p. 335).

Furthermore, $(y_K = \alpha k; y_L = \beta n)$ stand for capital and labor's potential relative contribution to production growth. If $(y = 0)$, then $\frac{k}{n} = \left|-\frac{1-\alpha}{\alpha}\right|$ and $\mu = \frac{k}{n}$ so that:

$$(10)\ \mu = \frac{1 - \alpha}{\alpha}$$

In this, (μ) is the elasticity of resource composition, from which α can be estimated as:

$$(11)\ \alpha = \frac{1}{\mu + 1}$$

α is the elasticity of capital contribution to production growth (Villalobos, 2019a). Inserting equation (8) into equation (9) will result in Swan's basic formula for the relative rate of growth of output:

$$(12)\ y = \alpha s\frac{Y}{K} + \beta n$$

Swan's rate of profit (r), also equivalent to the rate of interest *driven by the forces of perfect competition*, is:

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[3] See Swan's (1956, p. 335) footnote 3 for this reference.

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$(13)\ r = \alpha \dfrac{Y}{K}$

Hence, the rate of profit is a fraction (α) of the average output of capital, which defines the marginal productivity of capital. Thus, the rate of profit is an amount equal to the marginal productivity of capital, meaning profit is measured in terms of a certain amount of output. Also, the *real* wage rate (w) is supposed in such "forces of perfect competition" as equivalent to marginal productivity of labor:

$(14)\ w = \beta \dfrac{Y}{L}$

For this definition, a profit rate for capital invested in labor must be yielded, which was not suggested by Swan. From equation (8), $\left(\dfrac{k}{s} = \dfrac{Y}{K}\right)$ which when placed in equation (13) will give:

$(15)\ r = \alpha \dfrac{k}{s}$

So, at a given (α), the rate of profit will depend on the rate at which savings satisfy the augment on capital accumulation. In that equation, $(r = y)$ as (k) grows proportional to (s) at *instantaneous* α. This brings Piketty's fundamental force for divergence to mind: $(r > y)$, which is his crucial assumption (Piketty, 2014, p. 25), but dubious (Krugman, 2015; Mankiw, 2015; Raval, 2017; Villalobos, 2019b; Galbraith, 2014). If $\emptyset = \dfrac{K}{Y} = \dfrac{s}{k}$, the prior equation or (13) gives rise to Piketty's tautological formulation $r = \dfrac{\alpha}{\emptyset} \therefore \alpha = r\emptyset$ or his first fundamental law of capitalism (Piketty, 2014, p. 52). Equation (15) can be expressed as $(sr = \alpha k)$, and Swan's basic formula given by equation (9) can be written as:

$(16)\ y = sr + \beta n$

This is what Swan (1956, p. 335) suggested in footnote 5. Suppose $y = 0$ so that per equation (16), $sr = -\beta n \therefore sr = -(1-\alpha)n \therefore \left|-\dfrac{sr}{n}\right| = (1-\alpha) \leftrightarrow \left|-\dfrac{\alpha k}{n}\right| = (1-\alpha)$ from which equation (10) will result.

Expanding Swan's *Unclassical* Case: Resource Productivity

Let $(\check{r})$ stand for the average rate of return on capital or capital income at a given instant; $\check{r} = \dfrac{Y}{K} - \alpha\dfrac{Y}{K}$ and thus:

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$$(17) \; \check{r} = (1-\alpha)\frac{Y}{K}$$

If $(\alpha = 1; \check{r} = 0)$ and $\beta = 0$. Similar for the labor productivity, $\hat{r} = \frac{Y}{L} - \beta\frac{Y}{L}$ ∴

$$(18) \; \hat{r} = (1-\beta)\frac{Y}{L}$$

$\hat{r}$ denotes the average rate of return on labor alone. By inserting equation (17) into (13), and equation (18) into (14) and after some movements, we obtain respectively:

$$(19) \; r = \check{r}\frac{\alpha}{1-\alpha} \; ; (0 \le \alpha < 1)$$

$$(20) \; w = \hat{r}\frac{\beta}{1-\beta}; (0 \le \beta < 1)$$

Equation (19) depicts the curve of the rate of profit of capital alone, depending on capital return and capital productivity, whose slope is traced by $\frac{\alpha}{1-\alpha}$. Equation (20) delineates the curve of wage rate of labor alone with slope $\frac{\beta}{1-\beta}$.

These two previous rates lead to a *balance* in capital and labor shares on production growth at each state of the arts. Replacing those equations and equation (8) into (9), we transform Swan's basic formula given by equation (12) into the following:

$$(21) \; y = (1-\alpha)\frac{r}{\check{r}}k + (1-\beta)\frac{w}{\hat{r}}n$$

This formula could explain oscillations in the share of income in the long run, provided that the state of the arts, market conditions, and government policies change. Assuming $(y = 0)$, the level of output will remain constant, and so $\frac{k}{n} = \left| -\frac{1-\beta}{1-\alpha}\frac{\check{r}}{\hat{r}}\frac{w}{r} \right|$ ∴

$$(22) \; \frac{k}{n} = \left| -\frac{\alpha}{1-\alpha}\frac{\check{r}}{\hat{r}}\frac{w}{r} \right| \; ; (0 \le \alpha < 1)$$

Resource composition plays its role in tracing the line in which production growth, cost growth, and profit lie (Villalobos, 2019a). Equalizing equations (17) and (18) will result in $\frac{K}{L} = \frac{1-\alpha}{1-\beta}$. Let $\kappa = \frac{K}{L}$ stand for resource composition, so that $\kappa = \frac{1-\alpha}{\alpha}$ and thus, $\alpha = \frac{1}{\kappa+1}$; but in this case, α measures capital contribution to the cost growth of production, for which we label it $\dot{\alpha}$ so that:

$$(23) \; \dot{\alpha} = \frac{1}{\kappa + 1}$$

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which with respect to equation (11), $(\mu = \kappa;\ \alpha = \grave{\alpha})$, that is, relative capital contribution to production growth equals relative capital contribution to the cost growth (Villalobos, 2019a).

Once $\kappa = \frac{1-\alpha}{\alpha}$ is placed into equation (22), it will yield:

$$(24)\ \mu = \left| -\frac{1}{\kappa}\frac{\check{r}}{\hat{r}}\frac{w}{r} \right|$$

μ stands for resource composition elasticity composited by resource composition and capital and labor relative rates of profit. After inserting $\kappa = \frac{1-\alpha}{\alpha}$ and equation (24) into (21) and making $(y = 0)$, we have $\frac{Y}{K} = \frac{1}{s}\mu n$ so that the average productivity of capital is:[4]

$$(25)\ y_K = \frac{1}{s}\mu n$$

Due to $\left(\frac{K}{L} = \kappa \therefore K = \kappa L \right)$, average labor productivity will be computed after being replaced in $\frac{Y}{K} = \frac{1}{s}\mu n$. Thus:

$$(26)\ y_L = \frac{\kappa}{s}\mu n$$

There is no surprise in these results; the same can be concluded from equation (12), which is Swan's equation, after replacing $\beta = (1 - \alpha)$ and making $(y = 0)$. By the ratio of equations (25) and (26), the composition of potential resource productivity is:

$$(27)\ \kappa_y = \frac{y_L}{y_K}$$

After differentiating this equation, we will unveil the potential resource productivity effect attributed to changes in the state of the arts as Swan (1956) suggested, or by capital and labor exploitation, complemented by government policies. This is:

$$(28)\ \acute{\kappa}_\gamma = \gamma_L - \gamma_K$$

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[4] When using $\grave{\alpha}$ instead of α, we arrive at the relative function of the cost of production and at the average capital and labor cost of production.

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So, (γ_K, γ_L) identify the relative average rates of capital and labor potential productivity.[5] If in equation (28) we suppose $(\gamma_K = \alpha\gamma, \gamma_L = \beta\gamma)$, it can transform into:

(29) $\acute{\kappa}_\gamma = \beta\gamma - \alpha\gamma$

By the derivative of $\kappa = \frac{K}{L}$, the relative rate of resource composition $\acute{\kappa}$ is found, denoting any fluctuations on resource composition. So, $d\kappa = d\left(\frac{K}{L}\right) \therefore$

(30) $\acute{\kappa} = k - n$

If $(k = n; \acute{\kappa} = 0, \mu = 1)$, then (κ, μ) remains the same at each instant, but not necessarily the state of the arts. Equation (28) shows that if $\gamma_K > \gamma_L$, then $\acute{\kappa}_\gamma < 0$. However, equation (30) denotes that at $(k > n)$, $\acute{\kappa} > 0$. As a hypothesis, the economy could promote increments in (K) provided that it entails a greater productivity of labor, which could reveal technological changes. These results explain that technological productivity $(\acute{\kappa}_\gamma)$ is implicit in resource productivity. It is a net result in economic growth: neither a simple "beyond the contribution of capital and labour" as Swan (1956, p. 337) wrote nor just a *residual factor* as Solow M. R. (1957) and Arrow, Chenery, Minhas, and Solow (1961) fruitlessly advocated.

If $(\gamma_K = \alpha\gamma; \gamma_L = \beta\gamma)$, a function of resource productivity growth can be defined as:

(31) $\gamma = \alpha\gamma + \beta\gamma$

With respect to equation (9), $(\alpha\gamma \neq \alpha k, \beta\gamma \neq \beta k)$, implying that resource contribution could differ from resource productivity along time; it is what we call *residue* (ε). This result provides evidence that technological and exploitation effects on production growth could result from the changes in the state of the arts, market conditions, and government policies. To highlight this, $\acute{\kappa}_\gamma$ is the relative net rate of productivity, which could be attributed to technological changes in capital and labor or exploitation productivity effects on the relative rate of economic growth. However, it could also be a sign of inefficient and ineffective resource allocation in the economy, unfair production growth distribution, or even a result of imperfect statistics. This tentative hypothesis could be supported by $\acute{\kappa}$ differing from $\acute{\kappa}_\gamma$ and thus equation (29) does not reveal $(\acute{\kappa})$, the technological productivity of resource composition.

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[5] A similar outcome can be acquired from equation (1) by the definition of $\kappa = \frac{K}{L} \therefore K = \kappa L, L = \frac{1}{\kappa}K$ and after differentiating the first result, we obtain $y_K = F(\kappa^{\alpha-1}); y_L = F(\kappa)^\alpha$ and $\kappa_y = \frac{y_L}{y_K}$ which is equation (27). From this, after taking the respective derivative, equation (28) would emerge. Likewise, (μ) from equation (10) and (α) from equation (11) will be obtained from that departure point after totally differentiating.

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So, $ḱ_\gamma$ could explain the deviations around κ, ceteris paribus, denoting how resource productivity can differ with respect to their contributions; at $[(\kappa = 1; \mathbf{y_L} = \mathbf{y_K}); (k = n), (ḱ_\gamma = \mathbf{0})]$, (y, γ) keeps its pace, but if $[(\kappa > 1; \mathrm{y_L} > \mathrm{y_K}); (k = n), (ḱ_\gamma > 0)]$, (γ) growth must occur. Nevertheless, its distribution between capital and labor might be unequal, which will be reflected by a fall in β. When equations (31) and (9) are matched, $[(\alpha, \beta); \mu]$ remains, revealing that no fraction of production growth will evaporate, and resource contribution could disclose convergence with divergence and, conceivably, resource share inequality. As it must be clear in the preceding analysis, $ḱ_\gamma$ could differ from $ḱ$, which could explain how and why resource contribution could create distance from resource share.[6] It is so because "the rate of technical progress" [and] "the rate of growth of labor may not be independent of the rate of accumulation" (Swan, 1956, p. 338).

The formula that simplifies those processes of the previous analysis can be obtained by replacing $(k = ḱ + n)$ from equation (30) into equation (9) as follows:

$$(32)\ \gamma = \alpha(ḱ + n) + \beta n$$

This is equivalent to Swan's basic formula given in equation (12), and after simplifying will result in:
$$(33)\ \gamma = \alpha ḱ + (\alpha + \beta)n$$

Due to $(1 = \alpha + \beta)$, $\gamma = \alpha ḱ + n$ and by equation (30) and the fact that $n = \dfrac{k}{\mu}$, we obtain $ḱ = \dfrac{\mu - 1}{\mu} k$ so that:

$$(34)\ \gamma = \left[\frac{\mu\alpha + (1 - \alpha)}{\mu}\right] k$$

This is our fundamental equation, equivalent to those given in equations (9) and (16) as long as $(ḱ_\gamma \neq 0)$, which shows the essential long-term dynamics of economic growth and its cycles and trends. Thus, it could depict the convergence-divergence problem prevailing in economic growth theory (Villalobos, 2019b).

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[6] Equation (29) has some similarity with that offered by Swan (1956, p. 338) in footnote 7. Nevertheless, equalization of equations (31) and (9) after defining $\beta = (1 - \alpha)$ obtained equation (10), which provides the elasticity of resource composition, and the effect of $ḱ_\gamma$ will technically disappear. Nothing as "external economies" suggested by Swan (1956, p. 338) in footnote 8 must occur beyond $(1 = \alpha + \beta)$, due to the fact that convergence and divergence are both results of the same processes of resource contribution to production growth and resource share on production growth, involving technological changes.

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Measuring Resource Share on Production Growth

Partial derivative of equation (1) in terms of capital is $yY = f(\alpha K^{\alpha-1} L^{\beta}) \therefore yY = f\left[\alpha \frac{F(K^{\alpha} L^{\beta})}{K}\right]$ and letting $r = yY$ be the rate of capital income or profit, we get equation (13). Defining $R = rK$ as the total capital income, we will obtain the following from the prior equation:

(35) $R = \alpha Y$

In the case of labor, $yY = f(K^{\alpha} \beta L^{\beta-1}) \therefore yY = f\left[\beta \frac{F(K^{\alpha} L^{\beta})}{L}\right]$ and making $w = yY$ to express the rate of labor income or wage, we get equation (14). If $W = wL$ is the total income (wage) for labor, then:

(36) $W = \beta Y$

In terms of distribution of production as capital and labor income, we obtain:

(37) $Y = R + W$

The same results can be derived from equation (9) due to $(y_K = \alpha k; \ y_L = \beta n)$ where $\alpha = \frac{y_K}{k}$; $\beta = \frac{y_L}{n}$ are, respectively, the elasticity of capital and labor productivity. By multiplying both sides of these expressions by (Y), we compute the amount of product or income that those resource contributions generate, as defined by equations (35) and (36).

Additionally, the relative share in production growth of those resources can be discovered by multiplying the above definitions by (γ), denoting relative capital and labor incomes in terms of its productivity:

(38) $\dot{r} = \alpha \gamma$
(39) $\omega = \beta \gamma$

By multiplying both terms of the prior equations by (Y), the average income in terms of productivity for capitalists and workers is obtained:

(40) $r = \alpha \gamma Y$
(41) $\dot{w} = \beta \gamma Y$

The average capital and labor share on production as income at each instant is:

(42) $\bar{Y} = r + \dot{w}$

Saving and Capital Accumulation

Equation (8) can be reformulated into $(K = sY)$, where (K) represents the stock of capital at each instant. The derivative of this definition can measure the rate of capital accumulation: $kK = syY$ so $\alpha k = sy$; and the relative rate of saving is:

$$(43) \ s = \alpha \frac{k}{y}$$

Thus, $k = \frac{1}{\alpha} sy$ which substituted in equation (34) will yield an equivalent equation of equation (43). This would be capable of explaining why the relative rate of saving could vary and how capital accumulation could occur:

$$(44) \ s = \alpha \frac{\gamma}{y} \left[\frac{\mu}{\mu\alpha + (1 - \alpha)} \right]$$

By multiplying both expressions of the previous equation by (Y), the amount of saving at each instant is deduced; $S = sY$:

$$(45) \ S = \alpha \frac{\gamma}{y} \left[\frac{\mu}{\mu\alpha + (1 - \alpha)} \right] Y$$

which is assumed to be equal to the quantity of capital accumulated at each instant. This capital accumulation includes consumption or *depreciation* $(\check{K})$, that fraction of (K) transferred to (Y), and new investments of capital. Now it is feasible to determine how much savings at each instant could come from capital and labor, respectively:

$$(46) \ S_K = \alpha \frac{\gamma}{y} \left[\frac{\mu}{\mu\alpha + (1 - \alpha)} \right] R$$

$$(47) \ S_L = \alpha \frac{\gamma}{y} \left[\frac{\mu}{\mu\alpha + (1 - \alpha)} \right] W$$

Swan's formula defined by equation (9) is equivalent to equation (34). If instead inserting equation (8) in it, we replace $k = \frac{1}{\alpha} sy$ derived from equation (43), so that we obtain:

$$(48) \ \gamma = sy + \beta n$$

Thus, Swan's equation (12) is ruled out because it does not provide a good measure of the influence of saving and capital accumulation on economic growth. Furthermore, if it were true

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that $\alpha k = sr$ as Swan suggested, then $k = \frac{1}{\alpha} sr$. After it is substituted into equation (9), it will yield equation (16), which is also equivalent to equation (48). This means that by equalizing equations (16) and (48), $(r = y)$. Nevertheless, in the case that (k) or (n) decline, it will be advantageous to use μ. Also, the *residue* (ε) can be calculated by $\varepsilon = (y - \gamma)$ or $sy = \alpha k$. $y_K = \alpha k$, so $sy = y_K$ and thus, $y = \frac{y_K}{s}$ so that:

$$(49)\ \varepsilon = \frac{y_K}{s} - \gamma$$

How can national savings be useful in financing the capital accumulation process? First, it must be clear that "(…) replacement falls far short of depreciation. Hence, investment net of depreciation cannot be identified with investment net of replacement" (Domar, 1953, p. 1-2). Let $\delta = \frac{\check{K}}{K} \therefore \check{K} = \delta K;\ \forall\{\delta:(0,1)\}$ be the rate of depreciation so that $\check{K}$ is the amount of capital effectively transferred to (Y), (κ) the remnant of capital, and к the new amount of capital investment. Ceteris paribus, the cost of production (C) could vary pursuant to (Y). Furthermore, к could appear as effective capital accumulation together with (κ) at each instant: $\kappa = K - \check{K} \therefore \kappa = K - \delta K \therefore$

$$(50)\ \kappa = (1 - \delta)K$$

So, at each instant (K) would reflect replacements into depreciation, and new investments which could imply technological progress.

A function of (K) can be described as:

$$(51)\ K = F(\kappa, \check{K})$$

At each instant, $1 = F\left(\frac{\kappa}{K}, \frac{\check{K}}{K}\right)$ and due to $\delta = \frac{\check{K}}{K}$, we deduce that $1 = F\left(\frac{\kappa}{K}, \delta\right)$ so that $(1 - \delta) = \frac{\kappa}{K} \therefore 1 = \delta + (1 - \delta)$. By differentiating equation (51) and inserting those definitions, the following is obtained:

$$(52)\ k = f[\delta \underline{k}, (1 - \delta)\underline{k}]$$

where $(\underline{k}, \underline{k})$ represent, respectively, the relative rates of depreciation and of the remnant of capital. Substituting equation (52) in equation (34) will reveal (γ). If replaced in equation (43), it will explain the reasons for the changes on (s), and so on and so forth.

If $k = 0$, then (K) remains the same quantity and $-\delta \underline{k} = (1 - \delta)\underline{k} \therefore \frac{\underline{k}}{\underline{k}} = \left|\frac{\delta}{1-\delta}\right|$ and if $\nu = \frac{\underline{k}}{\underline{k}}$ so:

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$$(53)\ v = \frac{\delta}{1-\delta}$$

(v) measures the elasticity of depreciation with which δ can be estimated:

$$(54)\ \delta = \frac{v}{1+v}$$

Given $ḳ = vḳ$, after being substituted in equation (52), it will yield $k = f[\delta ḳ, (1-\delta)vḳ] \therefore k = f[\delta, (1-\delta)v]ḳ$. Then, inserting equation (53), the result is the effective rate of capital accumulation governed by depreciation, which could include replacement and/or new capital of different technology at each instant (Villalobos, 2019a):

$$(55)\ k = 2\delta ḳ$$

An Addendum: Swan-Solow Hybrid Equation

Swan (1956), Solow M. (1956), and Arrow, J. K et al. (1961)'s economic growth analysis comes from the same point, saving and capital accumulation. Also, their assumptions are the same. Surprisingly, Solow's introduction of "A Contribution to the Theory of Economic Growth" starts by lessening Harrod and Domar's assumptions as "dubious" and the result of "suspect." Nevertheless, "the bulk of this paper is devoted to a model of long-run growth which accepts all the Harrod-Domar assumptions except that of fixed proportions," and at the end, he suggested introducing "'Keynesian relations and rigidities" (Solow M., 1956, p. 66).[7] Solow's fundamental equation (Solow M., 1956, p. 69) is:

$$(56)\ \dot{r} = sF(r, 1) - nr$$

By applying the reverse to this equation, we can explain the precedent results. Following Solow's definition, let $r = \frac{K}{L}$ instead of $\kappa = \frac{K}{L}$. When replaced inside $F(r, 1)$ in the prior equation, it will yield $\dot{r} = sF\left(\frac{K}{L}, 1\right) - nr$. The value [1] inside (F) represents $\left(\frac{L}{L}\right)$, which when inserted in the previous formula will result in $\left[\dot{r} = sF\left(\frac{K}{L}, \frac{L}{L}\right) - nr\right]$. Then, we can simplify so that $\left[\dot{r} = s\frac{F(K,L)}{L} - nr\right]$. We also know that $\left(L = \frac{K}{r}\right)$, so $\left[\dot{r} = rs\frac{F(K,L)}{K} - nr\right]$ and $Y = F(K, L)$, then $\left[\dot{r} = rs\frac{Y}{K} - nr\right]$.

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[7] Swan's "Economic Growth and Capital Accumulation" was issued in November 1956 and Solow's "A Contribution to the Theory of Economic Growth" in February 1956 as Swan (1956, p. 337) mentioned in footnote 6. Solow (1956) did not refer to Swan but in Arrow, J. K et al. (1961), Swan´s contribution to the theory of economic growth was recognized: "We note that Trevor Swan has independently deduced the constant-elasticity-of-substitution property of (ii). The function itself was used by Solow [I5], 77, as an illustration" (p. 230, in footnote 7)**.**

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Moreover, $\left[k = s\frac{Y}{K}\right]$ so $\left[\dot{r} = (k-n)\,r \therefore \frac{\dot{r}}{r} = k - n\right]$, and let $\left(\acute{\kappa} = \frac{\dot{r}}{r}\right)$ stand for the relative rate of growth of resource composition so that $\acute{\kappa} = k - n$, as equation (30) states. Yet more simply: dividing $\left[\dot{r} = rs\frac{Y}{K} - nr\right]$ by (r) so that $\left[\acute{\kappa} = s\frac{Y}{K} - n\right]$ and from here we can get equation (9).

Solow's fundamental equation expresses the path of resource composition: "Thus (6) states that the rate of change of the capital-labor ratio is the difference of two terms, one representing the increment of capital and one the increment of labor" (Solow M., 1956, p. 69). Swan's basic formula of economic growth given in equation (12) is based on the relative rate of growth of output, which is explicitly defined by resource contributions: profit and wage rates. But if $k = s\frac{Y}{K}$, we can return to Swan's equation (12) and after inserting equation (30), which is Solow's fundamental equation, we obtain equation (33); from this, a hybrid Swan-Solow equation of economic growth has been uncovered.

Evaluating the Developed Model: The Case of Costa Rica (2010 – 2017)

The purpose of this section is to provide an evaluation of the model we propose in this research by analyzing the case of Costa Rica. By examining the role of saving on capital accumulation and on economic growth during the period 2010 – 2017, we can reveal the robustness of the model. To achieve this result, it is necessary to measure resource contribution and share on production growth.[8]

A. **Data and Methodology**

Real values of annual Gross Domestic Product (Y) and of Gross Fixed Capital Formation (K) by the economy given at *market prices* and in millions of colons (1991=100) and computed until 2017 came from the Central Bank of Costa Rica (BCCR for its acronym in Spanish) (BCCR, 2019). The period of study begins in 2010 due to the changes in computing the number of workers – we call labor (L) – in the economy according to the National Institute of Statistical and Census (INEC for its acronym in Spanish) (INEC, 2019). In this analysis, we work with data of (L) provided at the IV-trimester of each year. Additionally, we built indices for each of those variables by taking their first year 2010 real value as a basis to measure the relative variations at each year during 1910 – 1917; to make it clear by taking labor as an example: $\frac{L_{2010}}{L_{2010}} * 100; \frac{L_{2011}}{L_{2010}} * 100 \dots \frac{L_{2017}}{L_{2010}} * 100$. These indicators are critical in the analysis of Costa Rica's resource contribution and share on economic growth and the role of saving and capital accumulation during those years.

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[8] Studies on production growth in the case of Costa Rica are based in the Growth Accounting Models; see: Abarca and Ramírez (2016), Álvarez (2018), and Robles (2019).

B. Analysis of Results

This section will describe and analyze the outcomes that the proposed model generates at each level when processing (Y, K, L) computed indices mentioned above. Table 1 registers the relative rates of variations of those effective indices $(\bar{y}, k, n)$ in that order and the resource composition (κ) at each instant. According to the formulas revealing the fundamental model, $(\mu, \acute{\kappa}, \acute{\alpha}, \alpha)$ is computed with those rates, which measure (γ) according to the fundamental equation (34) of economic growth and thus its potential level $(\bar{Y})$.

After the international economic crisis hit Costa Rica's economy in 2008 – 2010 (BCCR, 2010; BCCR 2011), it grew from $\bar{y}_{2011} = 0.042$ to $\bar{y}_{2012} = 0.052$, which was possibly influenced by a moderate international economic growth (2% – 3%), especially in the United States of America (the main trading partner of the country) and increments in national consumption and (K) (BCCR, 2012). Labor employment grew 2.3 percent points between 2011 – 2012 as capital increased by almost 100%, and $\bar{y}$ was slightly less than (γ). Business expectations of Costa Rican entrepreneurs led to $(k_{2013} = 0.123)$, a rate of 4.3 percent points compared to the prior year, led by industrial machinery and transport equipment especially in firms of the special regimen (BCCR, 2012), while $(n_{2013}) = 0.007$ percent points. But $\bar{y}_{2013} = 0.035$, approximately (-1.7) percent points over the previous year, influenced by a moderate increase in exportations and domestic consumption (BCCR, 2012).

In short, while $(\bar{y}_{2011} > \gamma_{2011})$ showing divergence between effective and potential production growth, $(\bar{y}_{2011} \cong \gamma_{2012})$ denoted convergence and $(\bar{y}_{2013} < \gamma_{2013})$ revealed divergence once again (Figures 1 and 2). In the subsequent years of the analyzed period, potential production growth diminished, possibly due to the strong decrement in employment. This implied a rise in effective production growth, which could be explained by greater labor exploitation rendered into greater labor productivity. This result could be caused by increments in the elasticity of resource composition (μ); the greater (μ), the lesser (γ) and the greater $\bar{y}$, as it appears have happened in 2011 and 2014 – 2016, when $\mu > 3.0$ (Figures 3 and 4); this variable fluctuates over the composition of resources in those years (Figure 5). This result could provide an answer the next question: "Can we estimate, within limits, whether this increase in production was purely fortuitous, whether it was primarily caused by technique, and the degree, if any, to which it responded to changes in the quantity of labor or capital?" (Cobb & Douglas, 1928, p. 139).

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Table 1

Costa Rica: Effective and Potential Production Growth (2010 – 2017)

Year	Y	L	K	κ	ȳ	k	n	μ	ќ	ὰ	α	γ	Ȳ
2010	100	100	100	1.00	-	-	-	-	-	-	-	-	100
2011	105	102	109	1.07	0.045	0.089	0.017	5.27	0.072	0.483	0.160	0.028	103
2012	110	106	118	1.11	0.052	0.080	0.040	2.01	0.040	0.473	0.333	0.053	108
2013	114	111	132	1.19	0.034	0.123	0.047	2.61	0.076	0.456	0.277	0.068	116
2014	118	109	138	1.26	0.035	0.045	-0.014	3.29	0.059	0.442	0.233	0.021	118
2015	121	107	149	1.39	0.028	0.082	-0.016	5.28	0.098	0.418	0.159	0.026	121
2016	126	109	158	1.44	0.042	0.055	0.018	3.12	0.037	0.410	0.243	0.027	124
2017	132	106	169	1.60	0.045	0.074	-0.033	2.24	0.106	0.385	0.308	0.045	130

Source: Elaborated by the author. Based on Central Bank of Costa Rica (BCCR, 2019).
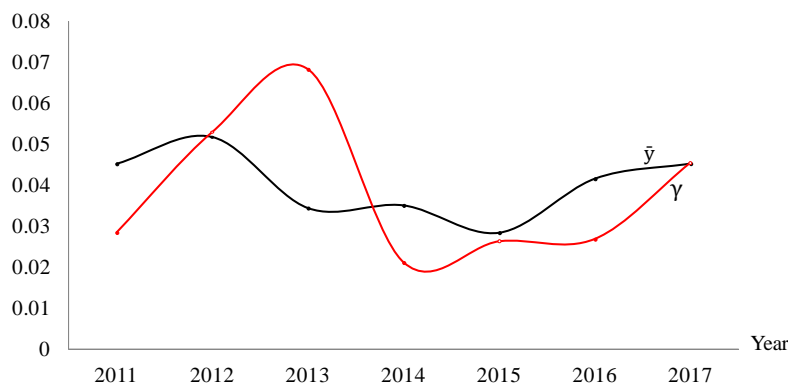

Figure 1. Costa Rica: Relative Rates of Effective and Potential Production Growth (2010 – 2017). Source: Based in Table 1.
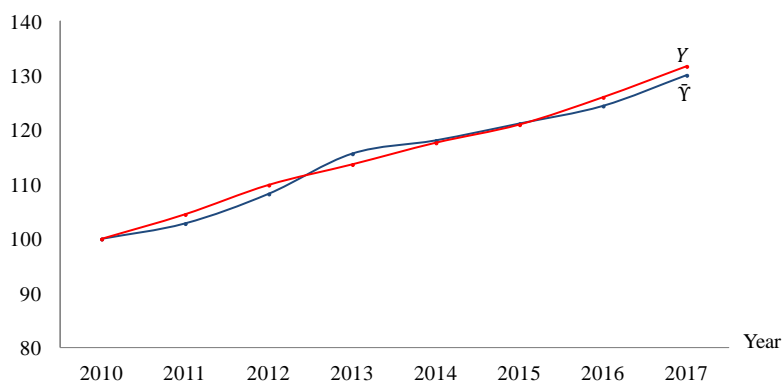

Figure 2. Costa Rica: Effective and Potential Production Growth Trends (2010 – 2017). Source: Based in Table 1.
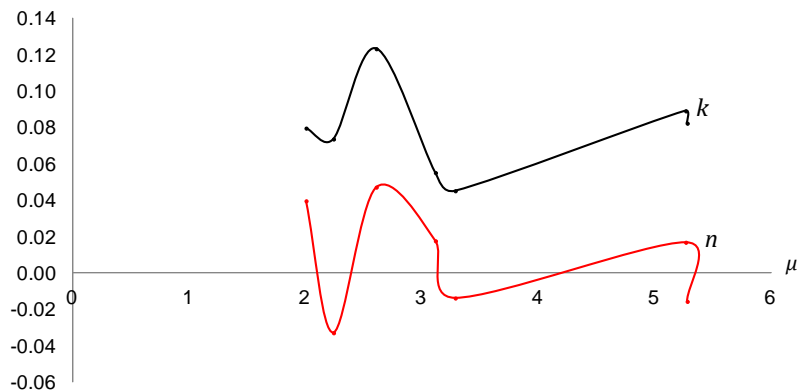
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Figure 3. Costa Rica: Relative Rates of Capital and Labor at Resource
Composition Level (2010 – 2017). Source: Based on Table 1.
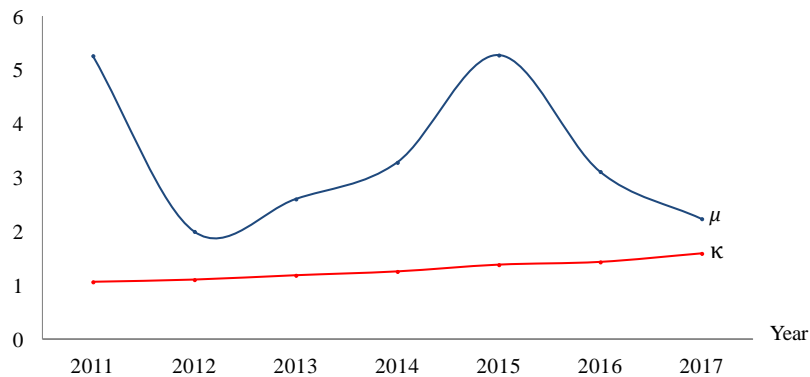

Figure 4. Costa Rica: Resource Composition and Elasticity of
Resource Composition (2010 – 2017). Source: Based on Table 1.

Divergences denoted by $(\bar{y}, \gamma)$ represent the *residue* ε as a result of the global production processes in the economy (Table 2, Figures 5 and 9). So, resource contribution to production growth converge $(y_K = y_L)$ but potential resource productivity diverge $(\gamma_K < \gamma_L)$ at any level of (μ); nevertheless, $(y = \gamma)$ (Figure 6):

> May it be possible to determine, again within limits, the *relative* influence upon production of labor as compared with capital? ... As the proportion of labor to capital changed from year to year, may be it possible to deduce the relative amount added to the total physical product by each unit of labor and capital and what is more important still by the *final* units of labor and capital in this respective year? (Cobb & Douglas, 1928, p. 139)

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But, effective labor share on production growth $\bar{y}_L$ is increasingly greater than capital share $\bar{y}_K$ as $\mu > 3.0$, in congruence with $(y_K = y_L)$; $(\gamma_K < \gamma_L)$ (Tables 2 and 3, Figures 7 and 8).

Regarding this issue, "…may we secure light upon the question as to whether or not the processes of distribution are modeled at all closely upon those of the production of value?" (Cobb & Douglas, 1928, p. 139-140). Average and total capital and labor share on effective production growth in Costa Rica during 2010 – 2017 could follow resource contribution and productivity (Table 4, Figures 10 and 11). Both average and gross capital income dropped during 2012 – 2015, when $(\bar{y} > \gamma)$ decreased as $(2 \leq \mu \leq 5.3)$ and $(n_{2014-15} < 0)$. Meanwhile, average labor fell less than the average capital income, but gross wage increased. Convergence appears once again showing an inverse relation between capital and labor gross income. Those results surged despite $(n_{2014-15} < 0)$ and perhaps due to an increment in real wage (BCCR, 2012; BCCR; 2015). As a general result, the greater (μ) is, the lower average capital income is as compared to labor income (Figure 12). Additionally, the lower the gross capital income is, the greater gross labor income becomes (Figure 13).

Table 2

Costa Rica: Relative Resource Contribution, Productivity, and Share on Production Growth and the Residue (2010 – 2017)

	A			B			C			(C - B) or (C - A)		
	Resource contribution			Resource productivity			Efective Resource share			Residue (ε)		
Year	y_K	y_L	y	γ_K	γ_L	γ	$\bar{y}_K$	$\bar{y}_L$	$\bar{y}$	$\bar{y}_K - \gamma_K$	$\bar{y}_L - \gamma_L$	$\bar{y} - \gamma$
2010	-	-	-	-	-	-	-	-	-	-	-	-
2011	0.014	0.014	0.028	0.005	0.024	0.028	0.007	0.038	0.045	0.0027	0.0141	0.0168
2012	0.026	0.026	0.053	0.018	0.035	0.053	0.017	0.034	0.052	-0.0004	-0.0008	-0.0013
2013	0.034	0.034	0.068	0.019	0.049	0.068	0.010	0.025	0.034	-0.0094	-0.0245	-0.0339
2014	0.011	0.011	0.021	0.005	0.016	0.021	0.008	0.027	0.035	0.0033	0.0107	0.0140
2015	0.013	0.013	0.026	0.004	0.022	0.018	0.005	0.024	0.028	0.0003	0.0018	0.0022
2016	0.013	0.013	0.027	0.007	0.020	0.027	0.010	0.031	0.042	0.0036	0.0112	0.0147
2017	0.023	0.023	0.045	0.014	0.031	0.045	0.014	0.031	0.045	-0.0001	-0.0001	-0.0002

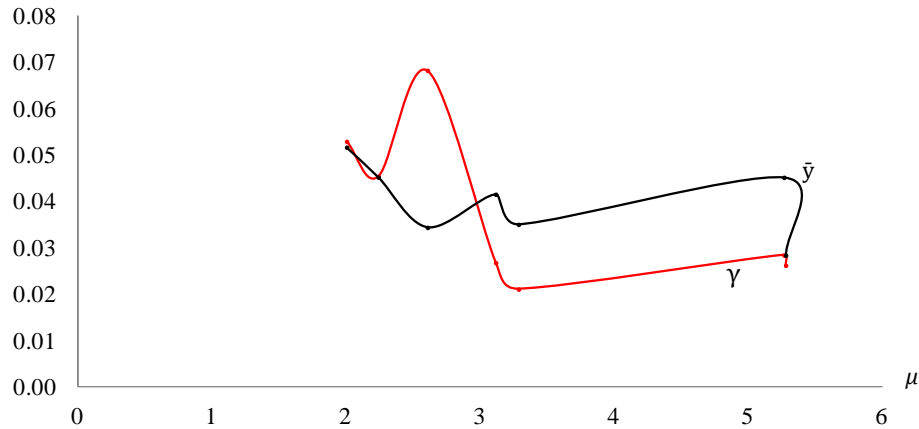Source: Based on Table 1.

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Figure 5. Costa Rica: Relative Rates of Effective and Potential Production Growth at Elasticity of Resource Composition (2010 – 2017). Source: Based on Table 1.
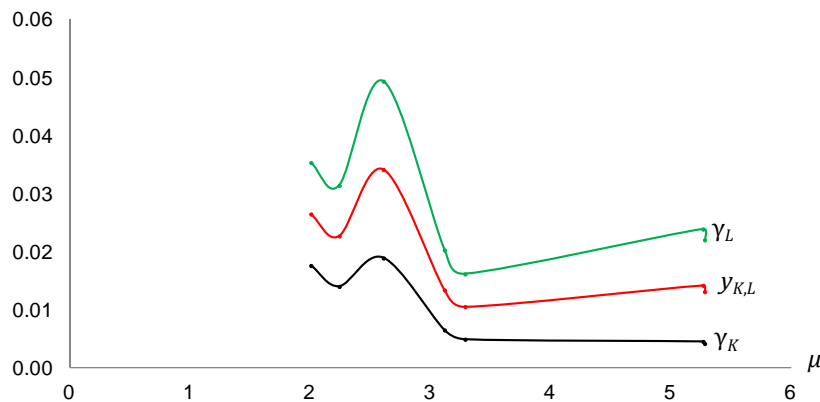

Figure 1. Costa Rica: Trends of Relative Rates of Capital and Labor Contribution to Production Growth and Productivity with respect to the Elasticity of Resource Composition (2010 – 2017). Source: Based on Table 2.
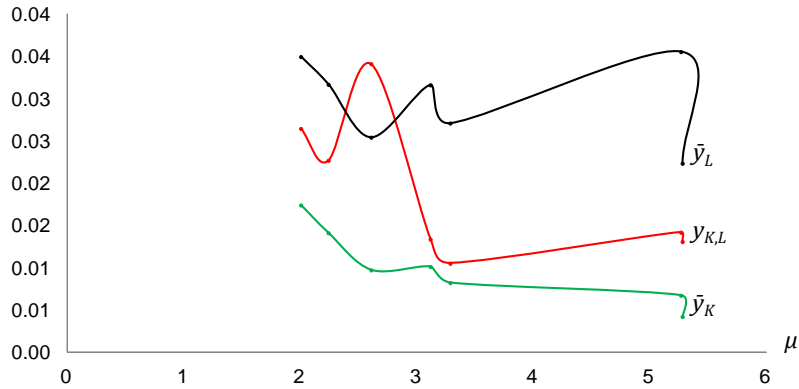
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Figure 7. Costa Rica: Trends of Relative Rates of Capital and Labor Contribution to Production Growth and their Effective Shares with respect to the Elasticity of Resource Composition (2010 − 2017). Source: Based on Table 2.
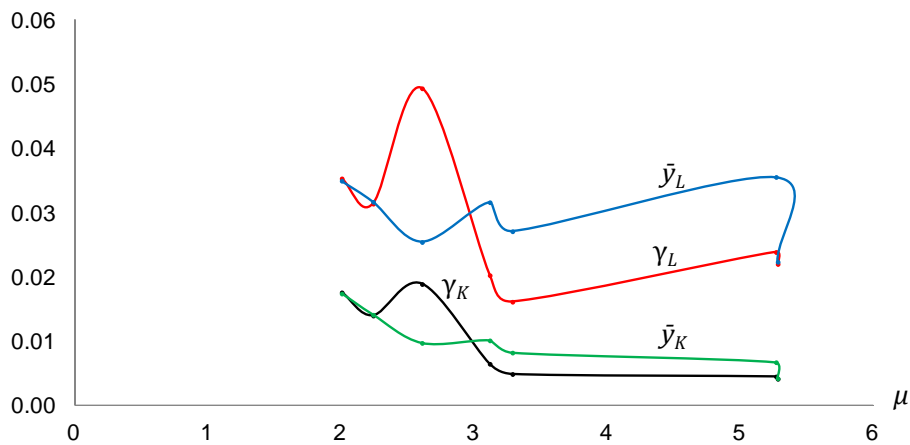

Figure 8. Costa Rica: Trends of Relative Rates of Capital and Labor Productivity and their Effective Shares on Production Growth with respect to the Elasticity of Resource Composition (2010 − 2017). Source: Based on Table 2.
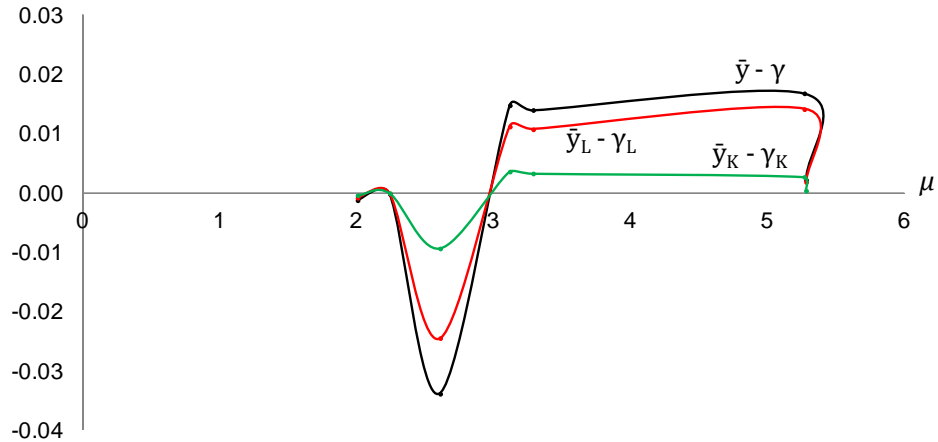
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Figure 2. Costa Rica: Trends of Relative Resource Residue at Elasticity of Resource Composition (2010 – 2017). Source: Based on Table 2.

Table 2

Costa Rica: Effective Relative Capital and Labor Productivity and Share on Production Growth (2010 - 2017)

Year	B			C		
	Resource productivity			Efective Resource share		
	γ_K	γ_L	$\acute{K}_\gamma$	$\bar{y}_K$	$\bar{y}_L$	$к_{\bar{y}}$
2010	-	-	-	-	-	-
2011	0.005	0.024	0.019	0.007	0.038	0.031
2012	0.018	0.035	0.018	0.017	0.034	0.017
2013	0.019	0.049	0.030	0.010	0.025	0.015
2014	0.005	0.016	0.011	0.008	0.027	0.019
2015	0.004	0.022	0.018	0.005	0.024	0.019
2016	0.007	0.020	0.014	0.010	0.031	0.021
2017	0.014	0.031	0.017	0.014	0.031	0.017

Source: Based on Table 2.

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Table 4

Costa Rica: Capital and Labor Share – or Income – on Production Growth (2010 – 2017)

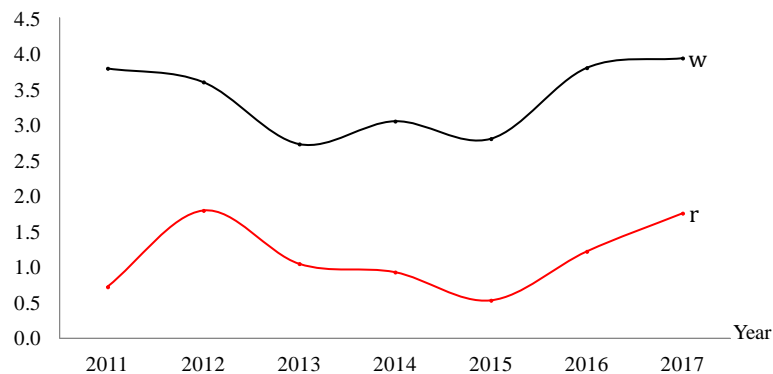| Year	r	w	λ	R	W	Y
2010	-	-	-	-	-	100
2011	0.7	3.8	4.5	17	88	105
2012	1.8	3.6	5.4	37	73	110
2013	1.0	2.7	3.8	31	82	114
2014	0.9	3.1	4.0	27	90	118
2015	0.5	2.8	3.3	19	102	121
2016	1.2	3.8	5.0	31	95	126
2017	1.8	3.9	5.7	41	91	132

Source: Based on Table 1.

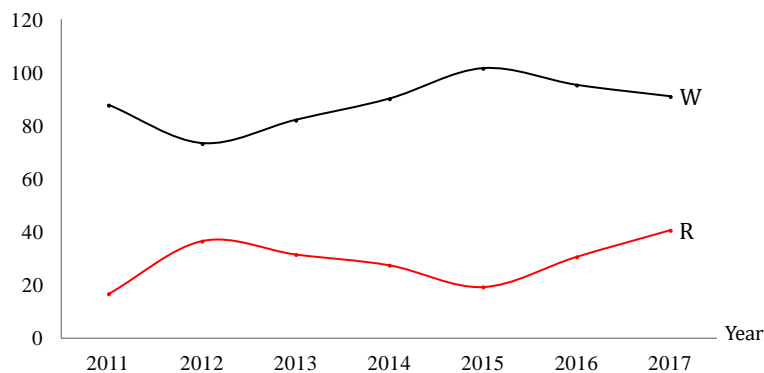*Figure 10.* Costa Rica: Trends of Average Capital and Labor Income (2010 – 2017). Source: Based on Table 4.

Figure 11. Costa Rica: Trends of Capital and Labor Gross Income (2010 – 2017). Source: Based on Table 4.
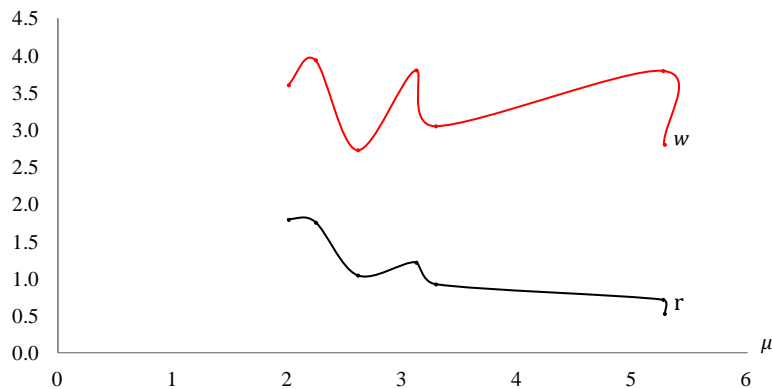
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Figure 123. Tendency of Average Capital and Labor Share at the Elasticity of Resource Composition (2010 – 2017). Source: Based on Table 4.
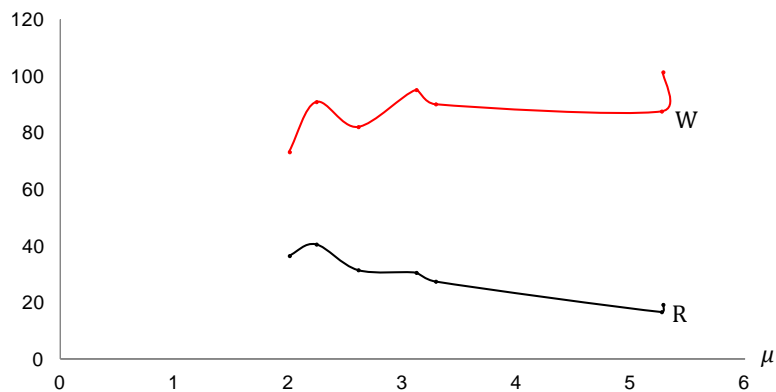

Figure 13. Costa Rica: Tendency of Total Capital and Labor Share at the Elasticity of Resource Composition (1910 – 1917). Source: Based on Table 4.

It is possible that the level of thrifts has been influencing the process of capital accumulation in Costa Rica during 2010 – 2017, but it is clear that it depends on foreign direct investment (BCCR, 2009, p. 10; BCCR, 2013, p. 13) and foreign savings (BCCR, 2010). As a result of the analysis of the effects of saving on capital accumulation and economic growth in Costa Rica, the model we propose determines that after a high rate of depreciation in 2011, it falls almost at the level of the rate of thrifts in the next year, dropping in 2012 (Table 5, Figures 15 and 16). In those years, (μ) precipitates from $\mu = 5.27$ to $\mu = 2$, and y_K falls while labor contributes twice as much compared to capital to production growth. (r) increases 2.7 times from 2011 to 2012 as long as average real labor incomes remained stagnant while its gross income (W) resulted in a 16% deterioration. In 2013, the rate of depreciation dropped 65% compared to the previous year, and it contrasted with the level of saving rate (92%) and capital accumulation (12.3%). In this year, μ

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grew at $\mu = 2.61$ due to fixed capital formation coming from new buildings in the manufacturing sector, and residential and industrial machinery and transport equipment, and the high level of savings came especially from direct foreign investment (BCCR, 2012). Labor productivity increased from $y_L = 0.035$ to $y_L = 0.049$, but its share on production growth diminished from $\bar{y}_L = 0.035$ to $\bar{y}_L = 0.025$, denoting a higher level of labor exploitation.

In the next years of the analyzed period, savings rates and capital accumulation returned to "normal" levels in the country (Table 5). In 2014, the rate of depreciation rose 3.18 times with respect to 2013, and μ jumped to higher levels because $(n < 0)$ and capital accumulation rose (Figures 14 and 15). $\mu_{2014} = 3.29$ and $\mu_{2015} = 5.28$ while $y_{L_{2014}} = 0.016$ and $y_{L_{2015}} = 0.022$ but the relative labor share was $\bar{y}_{L_{2014}} = 0.027$ and $\bar{y}_{L_{2015}} = 0.024$ in a frank deterioration in comparison to 2013. In 2015, the rate of depreciation fell to –13%, while capital accumulation grew 8.2% in consonance with the increase in savings rate, which could have a great influence in "…the beginning of the first stage of construction of the Moin Container Terminal and the execution of electricity generation and road infrastructure projects" (BCCR, 2015, p. 13)[9] which advanced the economy's impact during 2016 (BCCR, 2017). As a general result, if savings must be provided by the economy through labor and capital income, it is shown in Table 5 that it could represent a great sacrifice of national consumption (C).

Table 3
Costa Rica: Capital and Labor Contribution to Savings and Capital Accumulation: Capital Depreciation, Remnant and New Investment (2010 – 2017)

Year	s	S	$\check{K}$	δ	ķ	к	ķ	к	K	S_L	S_K	C_L	C_K	C
2010	-	-	-	-	-		-	-	100	-	-	-	-	-
2011	0.31	33	76	0.76	-	24	-	9	109	28	5	60	11	72
2012	0.51	56	61	0.56	-0.19	48	0.99	9	118	38	19	36	18	54
2013	0.99	113	19	0.16	-0.69	98	1.07	14	132	82	31	1	0	1
2014	0.30	35	103	0.78	4.35	29	-0.70	6	138	27	8	63	19	82
2015	0.46	56	93	0.68	-0.09	45	0.51	11	149	47	9	55	10	65
2016	0.32	41	117	0.78	0.25	32	-0.27	8	158	31	10	65	21	85
2017	0.50	66	103	0.65	-0.12	55	0.68	12	169	46	20	45	20	66

Source: Based on Tables 1 and 4.

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[9] "…el inicio de la primera etapa de construcción de la Terminal de Contenedores de Moín y la ejecución de proyectos de generación eléctrica e infraestructura vial" (BCCR, 2015, p. 13).
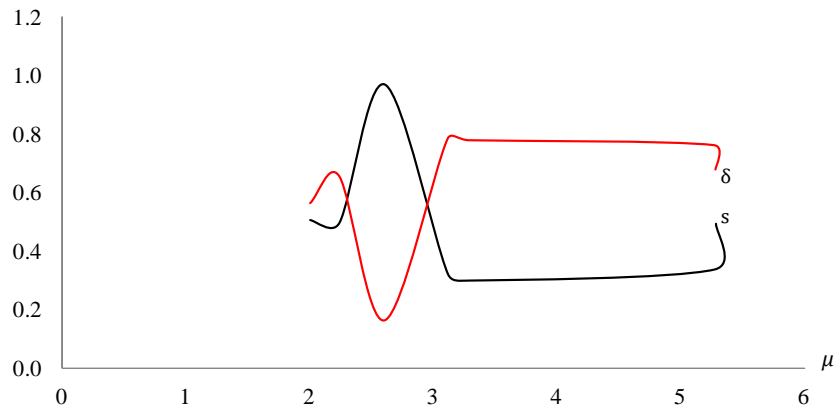
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Figure 14. Costa Rica: Trend of the Rate of Depreciation and Savings (2010 – 2017). Source: Based on Table 5.
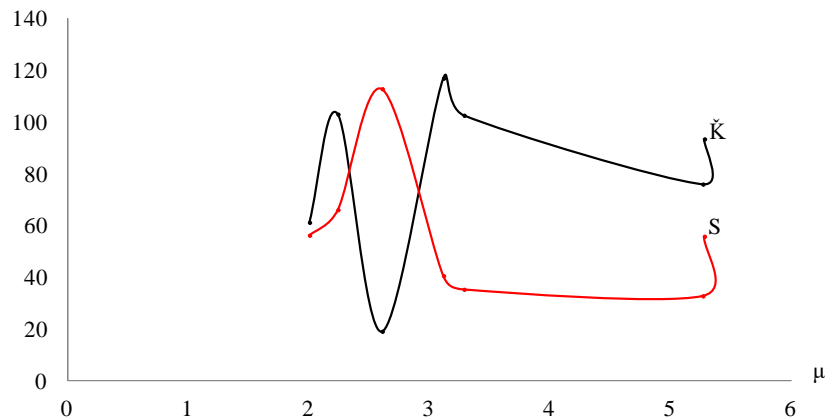

Figure 15. Costa Rica: Trend of the Levels of Depreciation and Savings (2010 – 2017). Source: Based on Table 5.

Conclusions

This research attempts to provide answers to Swan's (1956) hypothesis that there is a linkage between savings, capital accumulation, and growth of the productive labor force on economic growth. For this, Swan's (1956) crucial assumptions in his contemporary case were analyzed to pull out new formulas to measure the elasticity of resource composition or relative changes in the amount of labor in response to relative changes in the stock of capital and how this affects production growth. By expanding Swan's (1956) *unclassical* case, we unveiled how to quantify the average resource productivity, whose ratio determines an equivalent measure of resource composition. The results suggest that the forces of market do "drive the rate of profit or interest r and the (real) wage rate w into" [inequality] "with the marginal productivities of capital and labour" (Swan, p. 335).

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But there are endogenous forces governing production processes, in particular resource composition and its elasticity, reinforced by technological changes. These are responsible for convergence at the level of resource contribution to production growth with divergence at the level of resource productivity. Complementary, exogenous forces, commonly forces of market, government policies, and natural phenomena, could distort the structural divergence, inducing inequities between resource productivity and resource share. The evaluation of the model we developed in this research by analyzing Costa Rica's data reveals that the elasticity of resource composition is crucial in contrasting resource contribution to production growth and resource productivity with effective – *real* – resource share.

We found that resource productivity was greater than its contribution to production growth during 2010 – 2017, but it was greater for labor with respect to capital. Even though labor productivity was greater than its contribution to production growth at ($\mu > 2$), labor's share fluctuated closely around its contribution, and there are visible remains above labor productivity at ($\mu > 3$). Perhaps this is the reason why, between limits, in those years the average labor wage appeared to be somewhat higher than the average capital return, and why labor income increased sustainably while capital return declined. Among other results, Costa Rica appears to have had a clear relationship between savings and capital accumulation, including foreign savings as direct foreign investment, and the dynamic of economic growth. Of course, other forces exist like government policies and cycles of business in the country and in the world, especially linked to those main trading partners, which influenced those outcomes.

"Once the mind is accustomed to thinking in terms of trends of increase, the old static formulation of problems seems stale, flat and unprofitable" (Harrod, 1939, p. 15). We believe that we have contributed to finding answers to the theoretical problem of production growth contribution, productivity, and distribution. More interesting yet, this could provide answers to the problem of the residual factor (Solow, 1957) or to the suggestive phrase "beyond the contribution of capital and labour" (Swan, 1956). In our reasoning, it could indeed disclose the phenomenon of convergence and divergence and with it, the problem of inequality distribution of production growth between capitalists and workers and between workers (Stiglitz, 2015; Piketty, 2014; Krugman, 2015). Within limits, the main result of this research, never before done, is to have computed the relative, average, and gross capital and labor contribution to the economic growth and the rates of thrift, depreciation, and accumulation. Because of the limitations of data, we suggest that resources could not have been paid at their productivity, but in general "the processes of distribution are modeled at all closely upon those of the production of value" (Cobb & Douglas, 1928).

This endeavor could be useful on an academic scale, especially in economic science as well as in government policies and firms.

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References

Abarca, A., & Ramírez, S. (2016). *Estudio del Crecimiento Económico Costarricense, 1960-2014*. Retrieved from: http://odd.ucr.ac.cr/estudio-del-crecimiento-economico-costarricense-1960-2014/

Álvarez, C. (2018). *Estimación de una función de producción para Costa Rica: 1982 - 2017*. Retrieved from https://repositorioinvestigaciones.bccr.fi.cr/handle/20.500.12506/125

Arrow, J. K., Chenery, B. H., Minhas, S. B., & Solow, M. R. (1961). Capital-Labor Substitution and Economic Efficiency. *The Review of Economics and Statistics, 3*(43), 225-250. Doi: https://doi.org/10.2307/1927286

Banco Central de Costa Rica. (2009). *Programa Macroeconómico 2009-2010*. Retrieved from: https://www.bccr.fi.cr/seccion-publicaciones/publicaciones/programas-macroecon%C3%B3micos

Banco Central de Costa Rica. (2010). *Programa Macroeconómico 2010-2011*. Retrieved from: https://www.bccr.fi.cr/seccion-publicaciones/publicaciones/programas-macroecon%C3%B3micos

Banco Central de Costa Rica. (2011). *Programa Macroeconómico 2011-2012*. Retrieved from:https://www.bccr.fi.cr/seccion-publicaciones/publicaciones/programas-macroecon%C3%B3micos

Banco Central de Costa Rica. (2012). *Programa Macroeconómico 2012-2013*. Retrieved from:https://www.bccr.fi.cr/seccion-publicaciones/publicaciones/programas-macroecon%C3%B3micos

Banco Central de Costa Rica. (2013). *Programa Macroeconómico 2013-2014*. Retrieved from:https://www.bccr.fi.cr/seccion-publicaciones/publicaciones/programas-macroecon%C3%B3micos

Banco Central de Costa Rica. (2015). *Programa Macroeconómico 2015-2016*. Retrieved from:https://www.bccr.fi.cr/seccion-publicaciones/publicaciones/programas-macroecon%C3%B3micos

Daniel Villalobos Céspedes

Banco Central de Costa Rica. (2017). *Programa Macroeconómico 2017-2018*. Retrieved from: https://www.bccr.fi.cr/seccion-publicaciones/publicaciones/programas-macroecon%C3%B3micos

Banco Central de Costa Rica. (2019). *Principales indicadores*. Retrieved from: https://www.bccr.fi.cr/SitePages/default.aspx

Cobb, W. C., & Douglas, H. P. (1928). A theory of production. *The American Economic Review, 1*(18), 139-165. Retrieved from: https://www.jstor.org/stable/1811556?seq=1

Domar, E. D. (1946). Capital Expansion, Rate of Growth, and Employment. *Econometrica, 2*(14), 137-147. Doi: https://doi.org/10.2307/1905364

Domar, E. D. (1953). Depreciation, Replacement and Growth. *The Economic Journal, 249*(63), 1-32. Doi: https://doi.org/10.2307/2226748

Galbraith, J. K. (2014). Kapital for the Twenty-First Century? In *Dissent Magazine*. Retrieved from: https://www.dissentmagazine.org/article/kapital-for-the-twenty-first-century

Harrod, R. F. (1939). An essay in dynamic theory. *The Economic Journal, 193* (49), 14-33. Doi: https://doi.org/10.2307/2225181

Harrod, R. F. (1960). Second essay in dynamic theory. *The Economic Journal, 178*(70), 277-293. Doi: https://doi.org/10.2307/2228728.

Instituto Nacional de Estadística y Censos. (2019). *Labor market statistics*. Retrieved from http://www.inec.go.cr/empleo

Keynes, J. M. (1936). The general theory of employment, interest, and money (First ed.). London: Macmillan & Co.

Krugman, P. (2015). Review: The Economics of Inequality' by Thomas Piketty. In *The New York Times*. Retrieved from: https://www.nytimes.com/2015/08/03/books/review-the-economics-of-inequality-by-thomas-piketty.html

Mankiw, N. G. (2015). Yes, r > g. So What? *American Economic Review, 5*(105), 43-47. Doi: https://doi.org/10.1257/aer.p20151059

Daniel Villalobos Céspedes

Piketty, T. (2014). Capital in the twenty-first century. London: The Belknap Press of Harvard University Press. Retrieved from: https://www.hup.harvard.edu/catalog.php?isbn=9780674430006

Raval, D. (2017). *What's Wrong with Capital in the Twenty-First Century's Model?* In H. Boushey, J. B. DeLong, & M. Steinbaum, After Piketty. The Agenda for Economics and Inequality (First ed, 760). London: Harvard University Press. Doi: https://doi.org/10.4159/9780674978195-005

Robles, E. (2019). *Crecimiento de la productividad total de los factores en Costa Rica e inestabilidad macroeconómica 2019-01*. Retrieved from: http://economia.fce.ucr.ac.cr/en/publicacion/crecimiento-de-la-productividad-total-de-los-factores-en-costa-rica-e-inestabilidad

Solow, M. R. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics, 1*(70), 65-94. Doi: https://doi.org/10.2307/1884513

Solow, M. R. (1957). Technical change and the aggregate production function. *The Review of Economics and Statistics, 3*(39), 312-320. Doi: https://doi.org/10.2307/1926047

Stiglitz, J. E. (2015). *New Theoretical Perspective on the Distribution of Income and Wealth Among Individuals: Part I. The Wealth Residual*. Retrieved from: https://www.nber.org/papers/w21189

Swan, T. (1956). Economic growth and capital accumulation. *Economic Record, 2*(32), 334-361. Doi: https://doi.org/10.1111/j.1475-4932.1956.tb00434.x

Villalobos, C. D. (2019a). Crecimiento económico: contribución y participación de los recursos. *Revista ABRA*, *59*(39), 9-36. Doi: https://doi.org/10.15359/abra.39-59.1

Villalobos, C. D. (2019b). Crecimiento económico: Convergencia y divergencia. *Economía y Sociedad, 55*(24), 1-30. Doi: https://doi.org/10.15359/eys.24-55.3

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