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An Analysis of Students' Mathematical Reasoning in Solving Probability Problems Judging from Learning Styles: The Converger

Análisis del razonamiento matemático de los estudiantes en la resolución de problemas de probabilidad a juzgar por los estilos de aprendizaje: el convergente

Análise do Raciocínio Matemático dos alunos na resolução de problemas de probabilidade a julgar pelos estilos de aprendizagem: O Convergente

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Abstract

[Background] Several challenges are encountered by students when attempting to solve probability problems, one of which relates to reasoning about the given problem scenario. This phenomenon may arise due to variations in students' learning styles. **[Objective]** Thus, the current study aimed to examine students' reasoning abilities when dealing with probability problems, focusing on those with converging learning styles. **[Method]** This study employed a qualitative method, involving two students with a converging learning style who were invited to solve reasoning tasks. Three instruments were utilized: learning style questionnaire, mathematical reasoning tasks, and semi-structured interviews. The research data was analyzed through data reduction and display. **[Results]** The study's findings revealed that, in solving probability problems, convergers (students with a converging learning style) primarily relied on imitative reasoning. These students demonstrated four patterns. First, they accurately and comprehensively articulated known information from a question and correctly identified the question being asked. Second, they possessed a conceptual understanding of probability but provided inaccurate explanations regarding arithmetic sequences. Although they could devise problem-solving strategies, their articulation of these strategies lacked precision. Third, they utilized the concept of probability and employed problem-solving strategies to solve the given problem and explain the steps involved in reaching the solution. Four, they derived answers from problem-solving strategies and drew accurate conclusions, often exhibiting a tendency to trust their answers and subsequently verify them. **[Conclusions]** Students with converging learning styles predominantly used imitative reasoning when solving probability problems.

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Keywords: mathematical reasoning, probability problem, learning styles

Resumen

[Antecedentes] Los estudiantes enfrentan varios desafíos al intentar resolver problemas de probabilidad, uno de los cuales se relaciona con el razonamiento sobre el escenario del problema dado. Este fenómeno puede surgir debido a variaciones en sus estilos de aprendizaje. **[Objetivo]** Por lo tanto, el presente estudio tuvo como objetivo examinar la capacidad de razonamiento de los estudiantes cuando se enfrentan a problemas de probabilidad, centrándose en aquellos con estilos de aprendizaje convergentes. **[Método]** Se empleó un método cualitativo, para el cual se invitó a dos estudiantes, con un estilo de aprendizaje convergente, a resolver tareas de razonamiento. En esta investigación se utilizaron tres instrumentos: cuestionario de estilo de aprendizaje, tareas de razonamiento matemático y entrevistas semiestructuradas. Los datos de la investigación se analizaron mediante la reducción y visualización de datos. **[Resultados]** Los hallazgos del estudio revelaron que, al resolver problemas de probabilidad, los convergentes (estudiantes con un estilo de aprendizaje convergente) aplicaron, principalmente, razonamiento imitativo, en el que mostraron una tendencia a: (1) articular de manera precisa y completa la información conocida de una pregunta e identificar correctamente la pregunta que se hace; (2) poseen una comprensión conceptual de la probabilidad, pero brindan explicaciones inexactas sobre las secuencias aritméticas. Aunque podían idear estrategias de resolución de problemas, su articulación carecía de precisión; (3) utilizar el concepto de probabilidad y emplear estrategias de resolución de problemas para resolver el problema dado y explicar los pasos necesarios para alcanzar la solución; (4) derivar respuestas a partir de estrategias de resolución de problemas y sacar conclusiones precisas, mostrando, a menudo, una tendencia a confiar en sus respuestas y, posteriormente, verificarlas. **[Conclusiones]** Los estudiantes con estilos de aprendizaje convergentes utilizaron predominantemente el razonamiento imitativo al resolver problemas de probabilidad.

Keywords: aprendiendo estilos; razonamiento matemático; problema de probabilidad

Resumo

[Antecedentes] Os alunos enfrentam vários desafios ao tentar resolver problemas de probabilidade, um dos quais está relacionado com o raciocínio sobre o cenário do problema em questão. Este fenômeno pode surgir devido a variações nos estilos de aprendizagem dos alunos. **[Objetivo]** Portanto, o presente estudo teve como objetivo examinar a capacidade de raciocínio dos alunos diante de problemas de probabilidade, com foco naqueles com estilos de aprendizagem convergentes. **[Método]** O presente estudo utilizou um método qualitativo, onde dois alunos com estilo de aprendizagem convergente foram convidados a resolver tarefas de raciocínio. Nesta pesquisa foram utilizados três instrumentos: questionário de estilo de aprendizagem, tarefas de raciocínio matemático e entrevistas semiestruturadas. Os dados da pesquisa foram analisados por meio de redução e visualização de dados. **[Resultados]** Os resultados do estudo revelaram que, na resolução de problemas de probabilidade, os convergentes (alunos com estilo de aprendizagem convergente) utilizaram principalmente o raciocínio imitativo, no qual apresentavam tendência a: (1) articular de forma precisa e completa as informações conhecidas de uma pergunta e identificar corretamente a pergunta que está sendo feita; (2) possuem uma compreensão conceitual de probabilidade, mas fornecem explicações imprecisas de sequências aritméticas. Embora pudessem conceber estratégias de resolução de problemas, a sua articulação carecia de precisão; (3) utilizar o conceito de probabilidade e empregar estratégias de resolução de problemas para resolver o problema em

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questão e explicar os passos necessários para chegar à solução; (4) obter respostas a partir de estratégias de resolução de problemas e tirar conclusões precisas, muitas vezes mostrando uma tendência a confiar nas suas respostas e, posteriormente, verificá-las. **[Conclusões]** Os alunos com estilos de aprendizagem convergentes usaram predominantemente o raciocínio imitativo ao resolver problemas de probabilidade.

Palavras-chave: raciocínio matemático; problema de probabilidade; aprendendo estilos.

Introduction

Mathematical reasoning skills are crucial for students to learn at all educational levels, including the elementary, secondary, and tertiary education. The challenges students encountered in mathematical learning primarily stem from a deficiency in their reasoning skills (Lithner, 2003, 2011, 2017). The prevalence of a teacher-centered learning approach has been identified as a significant factor contributing to students' diminished reasoning skills, thereby reducing the meaningfulness of mathematical concepts imparted to them (Morrison *et al*., 2020). Students' lack of active participation in the learning process can also be attributed to the teacher's poor awareness of effective learning strategies (Mata-Pereira & da Ponte, 2017). This ineffective learning process may lead to a stagnation in the development of students' mathematical reasoning skills.

One prevalent issue encountered in mathematics learning pertains to its inherent abstractness, leading to challenges for students in comprehending specific concepts (Adu-Gyamfi *et al*., 2012). In 2018, the Program for International Student Assessment (PISA) revealed Indonesian students' subpar performance in mathematical reasoning (OECD, 2018; Tohir, 2019). However, the 2022 PISA results showed that there was an increase in the Indonesian students' mathematical performance, coinciding with innovations by the Indonesian government made innovations, such as simplifying the curriculum and encouraging reasoning activities in the classroom (PISA, 2023; Idil, Gülen and Dönmez, 2024). The PISA findings are similar to those reported by the Trends in International Mathematics and Science Study (TIMMS), which indicated that the Indonesian students' reasoning skills are not uniformly cultivated at the national level. These two findings are consistent with our preliminary study results, which showed that Indonesian students had poor performance in solving probability problems.

Poor reasoning skills mean that students demonstrate difficulty in solving every day or contextual problems (Savard & Polotskaia, 2017). The cognitive processes involved in students' reasoning activities are initiated by Polya's problem-solving framework, which encompasses the following four key steps: understanding the problem, planning a solution, implementing a problem-solving strategy, and drawing a conclusion (Szabo *et al*., 2020). Our preliminary interview findings indicated that most students exhibited a reduced capacity to effectively model the problem based on the information provided within the problem statement. These students also showed a notable deficiency in reasoning during the preparatory and implementation phases of strategy development. This deficiency was particularly pronounced when students were confronted with probability problems. A decrease in the level of student engagement in

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solving probability problems has been observed to have a negative impact on their cognitive abilities.

Mathematical reasoning can be categorized into two distinct types: imitative reasoning and creative reasoning (Lithner, 2008). Imitative reasoning refers to the cognitive process of recalling and retaining previously encountered or instructed problems. Creative reasoning involves generating innovative solutions during problem-solving processes (Granberg & Olsson, 2015). Both types can be observed in the learning process when students engage in problem-solving activities (Lithner, 2003). These reasoning methods enable students to engage in activities such as analyzing, proving, evaluating, explaining, and drawing logical conclusions (Herbert & Pierce, 2012).

Previous research has explored reasoning from diverse perspectives. Reasoning is understood as a mental activity that leads to changes in the direction of thinking (Hackenberg & Lee, 2015; Ikram *et al*., 2020; Lee, M, Y. & Hackenberg, 2014). Johansson (2016) and Lithner (2017) established the necessity of creative reasoning for achieving high problem-solving scores. Putri, Mukhni, & Jazwinarti (2014) discovered a correlation between mathematical reasoning and problem-solving skills in the classroom. These studies have confirmed the relationship between mathematical reasoning and problem-solving. Thus, in the current study, these two aspects are inherently interconnected.

In the current era, students are expected to demonstrate integrated reasoning and problem-solving skills to address problems (Lithner, 2017). Problem-solving is a crucial component of student learning, facilitating the development of critical thinking skills in addressing both procedural and conceptual problems (Marufi *et al*., 2021, 2022). The importance of the Polya's steps in problem-solving activities should be highlighted (Kang, 2015; Szabo *et al*., 2020); these activities include understanding the problem, planning, solving, and reassessing the problem-solving processes.

Additional factors that influence students' reasoning processes include their motivation to learn, preferred learning style, and social interactions. Learning style affects students' mathematical reasoning skills in certain ways (Boesen *et al*., 2010). Learning styles may include psychological learning styles such as field-dependent and field-independent (Pitta-Pantazi & Christou, 2009) as well as tempo conceptual learning styles such as impulsive and reflective (Singer *et al*., 2017). However, there is limited research on the examination of mathematical reasoning associated with Kolb's learning styles, which encompass the converging, diverging, assimilating, and accommodating types.

The present study attempts to answer the following question: What reasoning processes do students undergo in solving probability problems in relation to the Kolb's learning styles? This study focused the analysis on the convergers' reasoning processes. Its findings are expected to yield several noteworthy contributions: (1) they should provide valuable insights for students on the importance of mathematical reasoning skills, critical thinking, logical thinking, and mathematical thinking in mathematical problem-solving; (2) they offer teachers certain ways to identify students' mathematical reasoning skills that can significantly impact their academic performance. Armed with this knowledge, educators can devise appropriate instructional strategies that could strengthen students' mathematical reasoning skills.

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Methodology

Research Design

The present study utilized a qualitative research method. The descriptive data consisted of written words derived from the researcher's observations of participants' behavior (Miles *et al*., 2014). This study also explored students' mathematical reasoning skills as they solved probability problems in mathematics. The study focused on students with the converging learning style (one of learning styles suggested by Kolb).

Subjects

This study involved eight twelfth-grade students who were about to take their final school exams. The subjects were selected to represent the four distinct learning style categories proposed by David Kolb: converging, diverging, assimilating, and accommodating. The criteria for selecting the subjects were determined by the breadth of the material covered in class. The students were chosen from a convenience sample of students accessible to the study team in terms of program, location, and scheduling. Furthermore, the study considered situations to select the participants, including students who had completed advanced probability material and had experience in working with calculus in analytics and graphics context. The study also required that their class teacher provide information regarding the students' communication skills. The objective was to choose subjects that could provide abundant information after solving probability problems. In this paper, the explanation will concentrate on two subjects who have a converging learning style (the converger). The pseudonyms assigned to the participants help analyze their work on designated tasks. The students are known in this paper as S1 (Subject 1) and S2 (Subject 2).

Research Instruments

Three instruments were utilized in this investigation. First, the Kolb's learning style questionnaire was adapted from previous

Table 1. *Research Instruments*

Instruments	Descriptions
Learning style questionnaire	Indicators: • Carrying out practical applications of ideas. • Solving problems and making decisions based on solutions to problems. • Responding to challenges as opportunities through logical, coherent, objective, and analytical thinking. • Viewing information through abstract conceptualization and actively processing it. • Preferring to experiment with new ideas and practical applications.
Mathematical reasoning tasks	**Task#1** A survey was conducted on 1000 high school students in East Luwu. The collected data revealed that 450 students own cell phones, 250 have motorbikes, and 100 own both. Determine the probability that a randomly selected student does not have a cell phone or motorbike. **Task#2** In a container with 8 blue marbles and 4 white marbles, when drawing 2 marbles at random, determine the probability of selecting 2 blue marbles, 1 blue marble and 1 white marble, and 2 white marbles.

Note: derived from research.

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research (Khozaei *et al*., 2022) and modified to meet the requirements of the study. Second, mathematical reasoning tasks were developed to acquire written data regarding students' mathematical reasoning in problem-solving. Before being administered, the test was validated by experts. Following expert validation and revision, the test was administered to the research participants. The test results were analyzed in predetermined stages. Third, semi-structured interviews were conducted to determine participants' mental activity when solving probability problems and assess the connection between students' test responses and interview data. The instruments used are as follows:

Data Collection

The research data were collected using three distinct methods over consecutive time periods: a survey, a test, and interviews. Student work on mathematical reasoning was used as a reference for conducting the interviews. Additionally, the outcomes of student work were compared to the outcomes of interviews. The validity of the data was determined by measuring the consistency between the student's work and the interviews' results. The interviews were conducted to further investigate participants' problem-solving test results.

The current study commenced with the administration of Kolb's learning style questionnaire to all students enrolled in the specified class. The returned questionnaires were analyzed in accordance with the specified guidelines. Eight students representing Kolb's four learning styles were chosen to complete mathematical reasoning tasks, followed by interviews with each participant. Interview data were presented as transcripts of conversations between the researcher and the participants. The data were analyzed to uncover the reasoning processes employed by the students during each problem-solving phase. The study also assessed the compatibility of the interview data with the outcomes of students' mathematical reasoning test in the context of solving probability problems. Aligned and mutually supportive data were collected to generate a research conclusion. In cases where data did not align or were not corroborated, they were considered as additional evidence.

Data Analysis

Data analysis was conducted based on the framework proposed by Miles *et al*., (2018). First, data reduction involved summarizing, selecting key elements, prioritizing important aspects, and identifying themes and patterns. Reducing data enhanced clarity and facilitated the collection of additional data by researchers. Then, the data were presented through concise descriptions, charts, categorical relationships, flowcharts, and similar visual representations. Data visualization facilitated the researchers' comprehension of past events and enabled them to strategize future work based on derived insights. Finally, data verification and conclusion drawing were conducted to ensure a smooth flow throughout the entire data analysis process.

Analysis and Results

Description of S1's Work on the First Reasoning Task

This section outlines the work performed by S1 in completing the first reasoning task related to mathematical probability.

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Figure 1. *S1's Understanding Regarding Task#1*

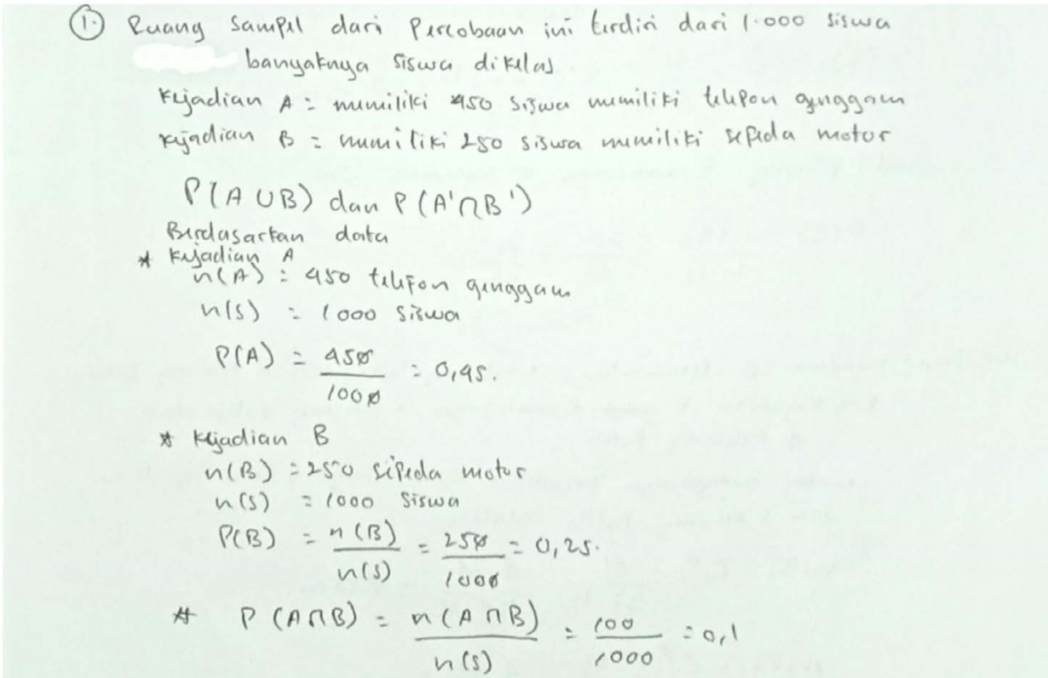

Figure 1 illustrates S1's comprehension of the task by accurately documenting all relevant information related to the probability of a compound event. In this instance, S1 recorded the available data, specifically that the experiment's sample comprises 1000 students, indicating the class size. S1 also assumed event A as "having a mobile phone" and event B as "having a motorcycle". In addition, S1 wrote the known information as $\boldsymbol{P(A \cup B)}$ ***and*** $\boldsymbol{P(A^c \cap B^c)}$. Then, S1 utilized mathematical calculations to determine the probability value of each event in the compound event scenario as $\boldsymbol{P(A) = \frac{450}{1000} = 0,45}$; $\boldsymbol{P(B) = \frac{250}{1000} = 0,25}$; and $\boldsymbol{P(A \cap B) = \frac{100}{1000} = 0,1}$. The work performed by S1 was reexamined and justified through an interview, an excerpt of which is provided below:

R: Did you understand the task?
S1: Yes, I did.
R: Can you explain the information you obtained from question number 1?
S1: It is about the sample size, students who have a mobile phone and students who have a motorcycle.
R: Was all the information useful to solve question number 1?
S1: Yes.

The interview excerpt demonstrates S1's comprehension of the assigned task. S1 could accurately and thoroughly explain and reference all the information provided in the assignment and utilize the provided information to address the problem.

After figuring out the probability of each event, $\boldsymbol{P(A)}$, $\boldsymbol{P(B)}$, and $\boldsymbol{P(A \cap B)}$, S1 wrote the strategy for solving the problem (Figure 2).

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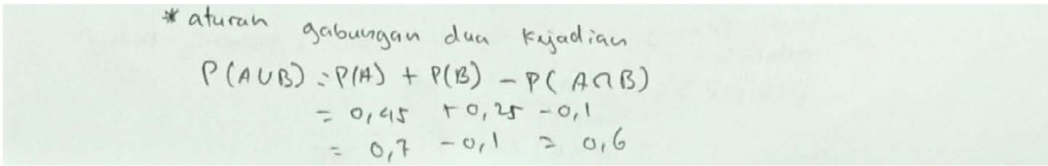
Figure 2. *S1's Strategy for Solving the Task#1*



In Figure 2, S1 selected and established the approach employed to accomplish the assigned task. S1 applied the probability rules to combine two events. Then, S1 proceeded with the execution of the selected strategy by providing a comprehensive written explanation as follows:

$$\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.45 + 0.25 - 0.1 = 0.6}.$$

Subsequently, S1 found the answer to the problem and drew a conclusion based on the characteristics of the events and the complement rule $\boldsymbol{P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - 0.6 = 0.4}$. Figure 3 depicts the complement rule used by S1 in solving the problem.

The S1's problem-solving processes were re-investigated and confirmed by the following interview excerpt:

R: What strategy did you use to solve the problem?
S1: determining the sample size and number of events.
R: Why did you use the strategy?
S1: Because that's the only strategy I know.
R: Do you usually use the strategy to solve similar problems?
S1: Yes. I use the same strategy to solve similar problems.
R: Can you explain how to use the strategy to solve the problem?
S1: We could use the strategy only if we knew the information to apply into the formula.

The interview excerpts suggest that S1 employed two strategies to accomplish the assigned task. First, S1 identified the sample size and, second, quantified the number of events. Based on the interview findings, it was observed that S1 had prior experience with employing this strategy to solve similar problems, leading S1 to opt for this approach when completing the assigned task.

Description of S2's Work on the First Task

This section describes the work conducted by S2 in completing the first reasoning task pertaining to mathematical probability.

Figure 3. *Rule of Complementary Events*

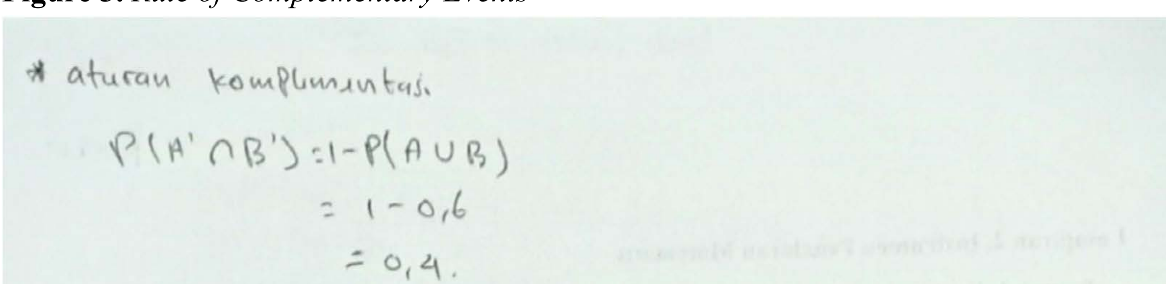


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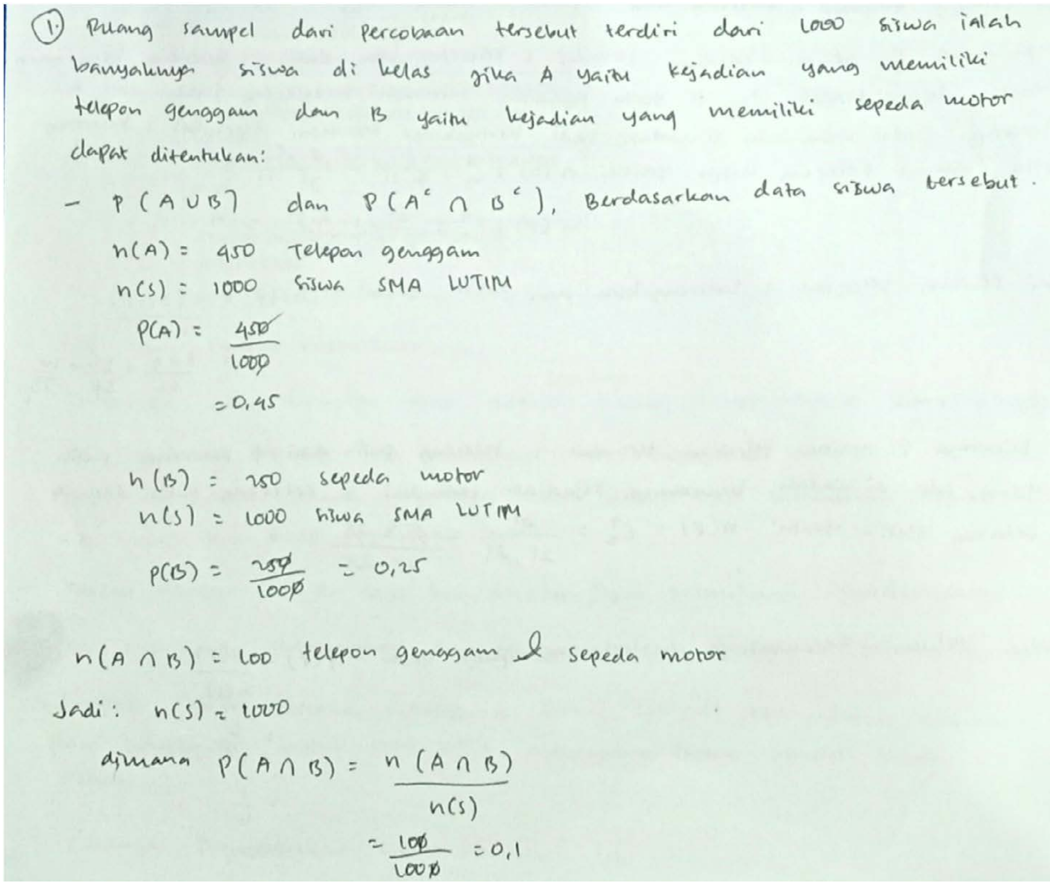
Figure 4. *S2's Understanding Regarding Task#1*



1. Ruang sampel dari percobaan tersebut terdiri dari 1000 siswa ialah banyaknya siswa di kelas jika A yaitu kejadian yang memiliki telepon genggam dan B yaitu kejadian yang memiliki sepeda motor dapat ditentukan:

- $P(A \cup B)$ dan $P(A^c \cap B^c)$, Berdasarkan data siswa tersebut.

$n(A) = 450$ Telepon genggam
$n(S) = 1000$ siswa SMA LUTIM
$P(A) = \frac{450}{1000} = 0{,}45$

$n(B) = 250$ sepeda motor
$n(S) = 1000$ siswa SMA LUTIM
$P(B) = \frac{250}{1000} = 0{,}25$

$n(A \cap B) = 100$ telepon genggam & sepeda motor
Jadi: $n(S) = 1000$
dimana $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{100}{1000} = 0{,}1$

Figure 4 illustrates S2's understanding of the task. In Figure 4, S2 displayed comprehensive information pertaining to the probability of a compound event. In this instance, S2 wrote the given information, stating that the sample space of the experiment comprises 1000 students. Then, S2 proposed an assumption that event A refers to "having a mobile phone" and event B refers to "having a motorcycle". Besides, S2 recorded the requested information in the following manner: $(\boldsymbol{A} \cup \boldsymbol{B})$ ***and*** $\boldsymbol{P}(\boldsymbol{A}^c \cap \boldsymbol{B}^c)$. Then, S2 explained that the probability problem of compound events could be solved using mathematical calculations to determine the probability value of each event: $\boldsymbol{P}(\boldsymbol{A}) = \frac{\mathbf{450}}{\mathbf{1000}} = \mathbf{0.45}$; $\boldsymbol{P}(\boldsymbol{B}) = \frac{\mathbf{250}}{\mathbf{1000}} = \mathbf{0.25}$; and $\boldsymbol{P}(\boldsymbol{A} \cap \boldsymbol{B}) = \frac{\mathbf{100}}{\mathbf{1000}} = \mathbf{0.1}$. This process was reexamined and proved by the following interview excerpt:

R: Did you understand the problem?
S2: Yes, I did
R: Can you explain the information you obtained from question number 1?
S2: sample size, data on each sample, the complement rule
R: Was all the information useful to solve question number 1?
S2: Yes

The interview excerpt suggests that S2 comprehended the assigned task, demostrating this by providing a detailed

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explanation of the task in their own words. S2 also demonstrated the capacity to utilize task-derived information.

After finding the probability of each event, which is $\boldsymbol{P(A)}, \boldsymbol{P(B)}$, and $\boldsymbol{P(A \cap B)}$, S2 wrote the strategy for solving the problem as presented in Figure 5.

Figure 5 illustrates that S2 selected and established two strategies for resolving the problem. The initial strategy involved the utilization of the probability rule for two combined events $\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$. The second strategy involved employing the characteristics of events and the complement rule, $\boldsymbol{P(A^c \cap B^c) = 1 - P(A \cup B)}$. Next, S2 continued the problem-solving process by implementing the selected strategy and documenting the formula comprehensively: $\boldsymbol{P(A \cup B) = 0.45 + 0.25 - 0.1 = 0.6}$. Following that, S2 employed the second strategy, which utilized the characteristics of the events and the complement rule to identify the solution to the problem and derive conclusions from the task as $\boldsymbol{P(A^c \cap B^c) = 1 - 0.6 = 0.4}$.

S2's problem-solving strategy underwent reexamination and was confirmed through an interview, an excerpt of which is provided below:

R: What strategy did you use to solve the problem?

S2: Determining the sample size and the appropriate formula to solve the problem.

R: Why did you use the strategy?

S2: Because by using the strategy, I could understand the solution.

R: Do you usually use the strategy to solve similar problems?

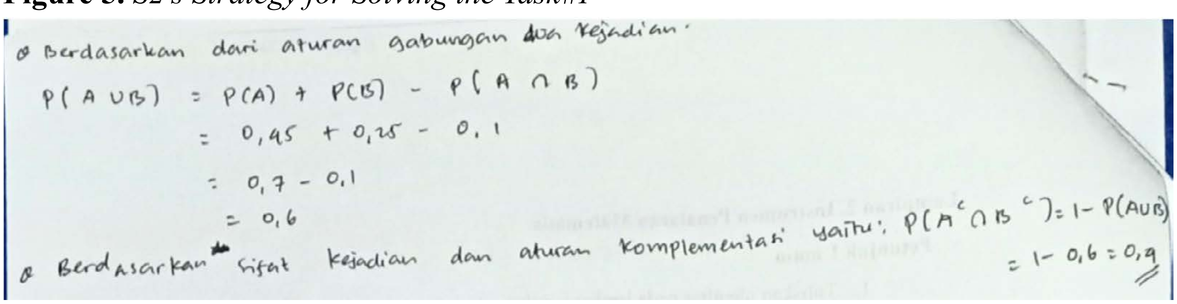
S2: Yes, I have applied the same strategy to similar problems, such as combined events, probability, and complement

R: Can you explain how to use the strategy to solve the problem?

S2: First, determining the sample size, then the probability rule for combined and complementary events and complement.

The provided interview excerpt affirms that S2 employed two strategies when accomplishing the assigned task. S2's statement supports the finding that determining the sample size and identifying applicable formulas are important factors. The fact about the problem-solving strategies used by S2 was supported by the following statement: "I have applied the same strategy to similar problems, such as combined events, probability, and complement". The interview results indicated that S2 was accustomed to employing the strategy when faced with similar problems. Consequently, S2 opted to utilize this method in solving the probability task, as it allowed for a rapid comprehension of the solution.

Figure 5. *S2's Strategy for Solving the Task#1*



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Description of S1's Work on the Second Task

This section outlines the efforts made by S1 in completing the second reasoning task related to mathematical probability.

Figure 6 depicts S1's comprehension of the second task, which pertains to the probability of a compound event. S1 proceeded to articulate their understanding in their own words. S1 wrote the information by calculating the sample size of the experiment, which included 12 marbles. Then, to find the sample size, S1 applied the following formula: $\boldsymbol{n(s) = C_2^{12} = \frac{12!}{10!2!} = \frac{12.11.10!}{10!.2.1!} = 66}$. S1 recognized that sample size was a crucial piece of information that could be utilized to derive the desired solution for the given task.

After that, S1 selected and determined a strategy to solve the problem. To calculate the probability of choosing two blue marbles, S1 used the following strategy shown in Figure 7.

Figure 7 illustrates S1's selection and determination of two strategies for task completion. Initially, S1 employed the formula for combinations $\boldsymbol{n(B) = C_2^8}$; and then used the probability formula $\boldsymbol{P(B) = \frac{n(B)}{n(S)}}$. However, before writing the strategies, S1 assumed event B (the probability of choosing two blue marbles). As there are eight blue

Figure 6. *S1's Understanding Regarding Task#2*

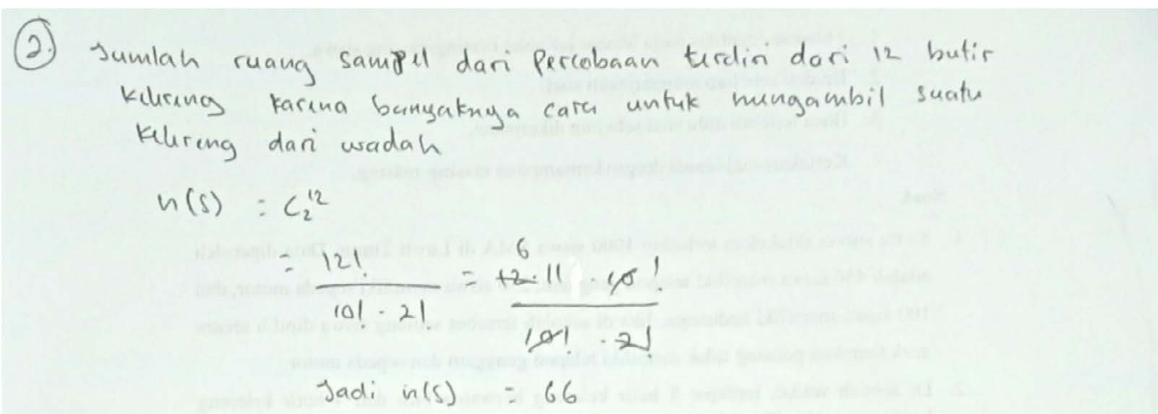
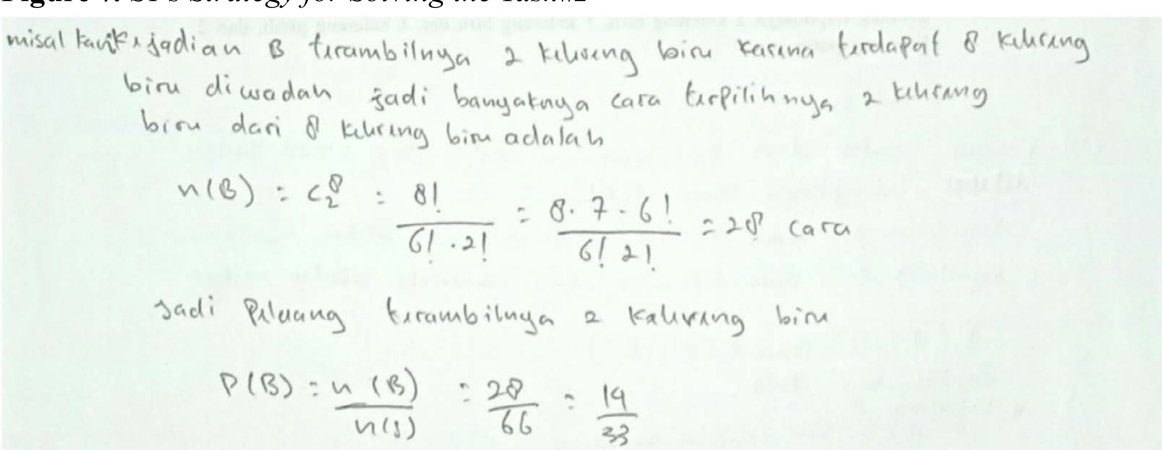


Figure 7. *S1's Strategy for Solving the Task#2*



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marbles available, S1 wrote $\boldsymbol{n(B) = C_2^8}$. Then, S1 implemented the strategy by writing $\boldsymbol{n(B) = \frac{8!}{6!2!} = \frac{8.7.6!}{6!.2.1!} = 28\ cara}$. Finally, S1 determined the answer to the problem and drew a conclusion by implementing the second strategy, which is $\boldsymbol{P(B) = \frac{28}{66} = \frac{14}{33}}$.

Then, S1 selected and determined the strategy for calculating the probability of selecting one blue marble and one white marble. Figure 8 illustrates the problem-solving strategy employed by S1.

In Figure 8, S1 selected and determined the strategy for calculating the probability of choosing one blue marble and one white marble by writing the following formula: $\boldsymbol{n(B) = C_1^8,\ n(P) = C_1^4,\ and\ P(B \cap P) = \frac{n(B \cap P)}{n(S)}}$. Nevertheless, before writing the strategies, S1 wrote two assumptions, where event B refers to the probability of choosing one blue marble out of eight blue marbles and event P refers to the probability of choosing one white marble out of four white marbles. After that, S1 applied the first and second strategies by writing them in detail as follows: $\boldsymbol{n(B) = \frac{8!}{7!1!} = \frac{8.7!}{7!.1!} = 8\ combinations}$, $\boldsymbol{n(P) = \frac{4!}{3!1!} = \frac{4.3!}{3!.1!} = 4\ combinations}$. Subsequently, S1 discovered the solution by using the strategy and drew a conclusion by implementing the third strategy which is $\boldsymbol{P(B \cap P) = \frac{8 \times 4}{66} = \frac{32}{66} = \frac{16}{33}}$.

Finally, S1 selected and determined the strategy for finding out the probability of selecting two white marbles. Figure 9 shows the strategy.

Figure 9 illustrates S1's strategy for calculating the probability of choosing two white marbles. The subject wrote two strategies in the form of the following formula: $\boldsymbol{n(P) = C_2^4\ and\ P(P) = \frac{n(P)}{n(S)}}$. However, prior to determining the strategies, S1 assumed that event P was the probability of choosing two white marbles out of four white marbles. Then, S1 applied the first problem-solving strategy by writing the formula $\boldsymbol{n(P) = \frac{4!}{2!2!} = \frac{4.3.2!}{2!.2!} = 6\ combinations}$.

Figure 8. *S1's Strategy for Solving the Task#2*

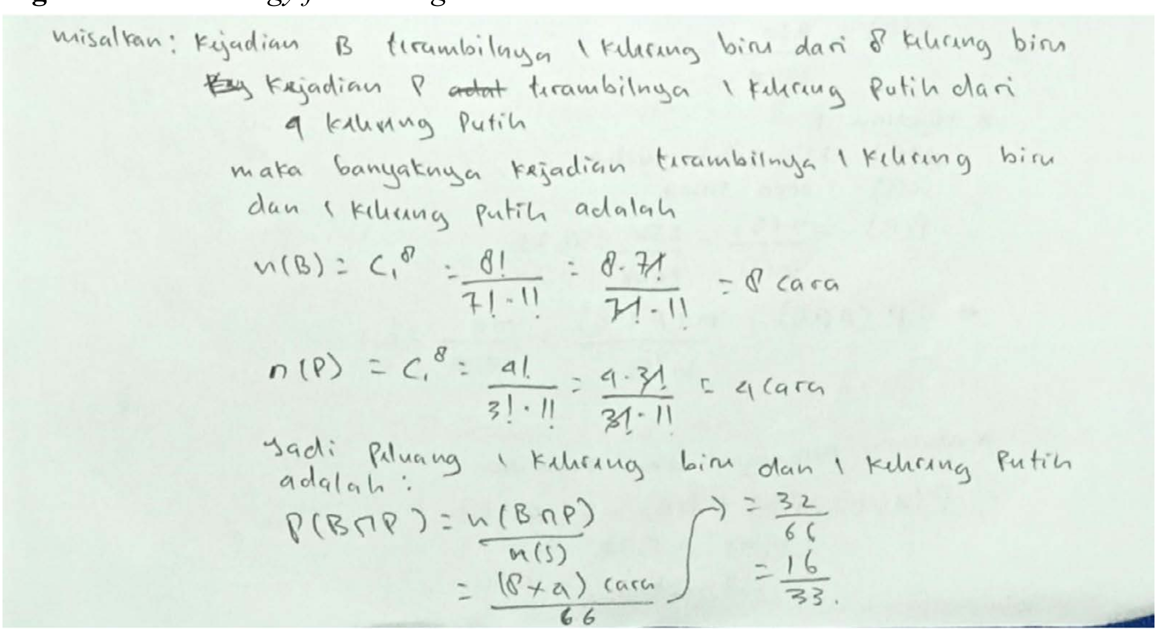

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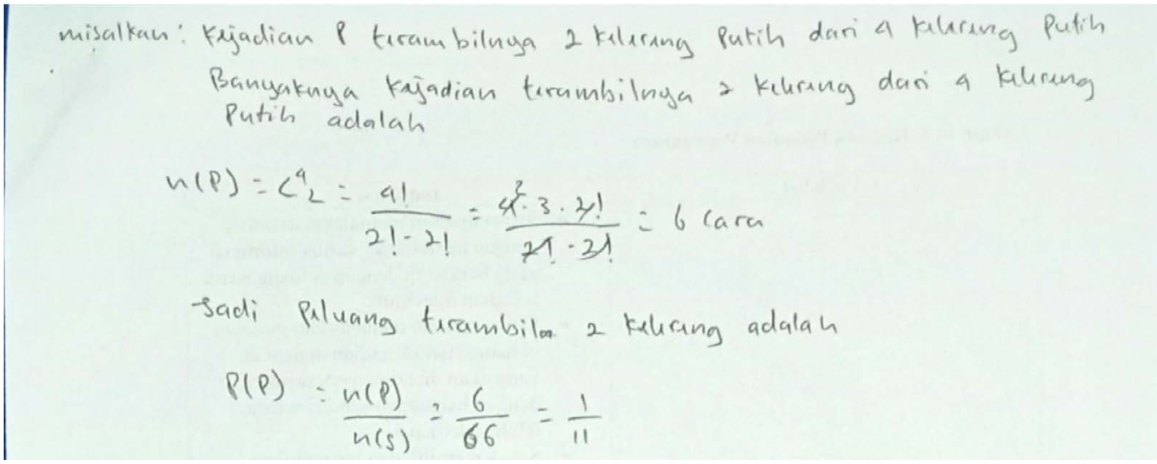
Figure 9. *S1's Strategy for Calculating the Probability of Choosing Two White Marbles*



At last, S1 figured out the answer and the conclusion to the task by implementing the second strategy, which is $\boldsymbol{P}(\boldsymbol{P}) = \frac{\mathbf{6}}{\mathbf{66}} = \frac{\mathbf{1}}{\mathbf{11}}$.

The problem-solving processes undertaken by S1 were reinvestigated and confirmed through an interview; here is an excerpt of it:

R: How did you solve question number 2?
S1: Using the probability formula
R: Is there another way that can be used to solve question number 2?
S1: Yes, there is, by using the formula for combinations.
R: What are the steps to solve a problem like this?
S1: I must understand the formula appropriate for solving a problem like this.

The interview excerpt above indicates that S1 used the probability formula to solve question number 2. Besides using the probability formula, S1 also mentioned the possibility of using the formula for combinations to solve the problem. In the interview excerpt, S1 stated that to solve the problem, he had to understand the appropriate formula.

We conducted a more in-depth interview to examine the conclusion made by S1 on the probability of a compound event.

R: Could you write the conclusion on the problem using your own words?
S1: Yes, I could
R: Are you sure about your answer?
S1: Yes, because I have used my reasoning skills
R: Have you examined the result?
S1: Yes, I have thoroughly reviewed the outcome.

The interview excerpt indicates that S1 could articulate responses in written form. Furthermore, S1 demonstrated confidence in the accuracy of their answers and diligently verified their work prior to submitting it to the teacher.

The Description of S2's Work on the Second Task

This section describes Subject 2's (S2) efforts in completing the second reasoning task pertaining to mathematical probability.

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Figure 10. *S2's Understanding Regarding Task#2*

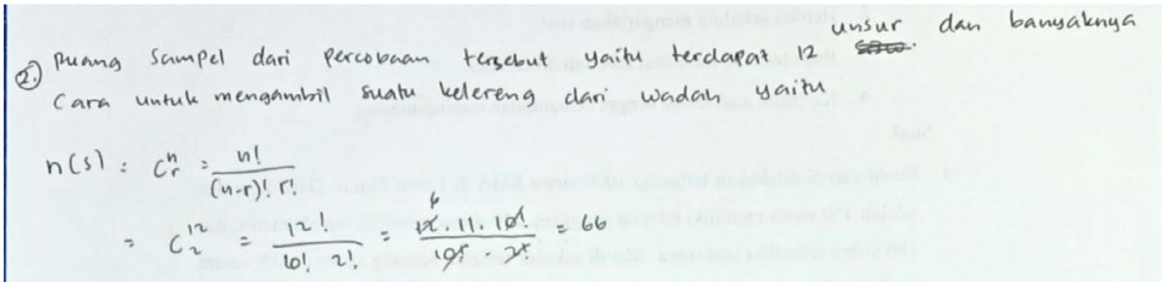

Figure 10 illustrates S2's understanding of the second reasoning task, where the subject could articulate the information on the probability of a compound event in written form. In this case, S2 wrote that the sample size consisted of 12 elements. Then, S2 calculated the sample size by involving the formula $\boldsymbol{n(s) = C_2^{12} = \frac{12!}{10!2!} = \frac{12.11.10!}{10!.2.1!} = 66}$. In short, S2 acknowledged the importance of this information in the problem-solving processes.

Then, S2 selected and determined the strategies for solving the problem. To calculate the probability of choosing two blue marbles, S2 used the following strategy.

In Figure 11, S2 selected and determined two strategies to solve the problem. First, he used the formula for combinations $\boldsymbol{n(B) = C_2^8}$. Then, he utilized the probability formula $\boldsymbol{P(B) = \frac{n(B)}{n(S)}}$. Nevertheless, before writing the two strategies, S2 made an assumption that event B was the probability of choosing two blue marbles (since there are eight marbles available). Then, S2 applied the following formula: $\boldsymbol{n(B) = C_2^8}$. After that, S2 explained the strategy by writing it in detail as follows: $\boldsymbol{n(B) = \frac{8!}{6!2!} = \frac{8.7.6!}{6!.2.1!} = 28\ combinations}$. Finally, S2 determined the answer and drew a conclusion using the second strategy by involving the mathematical process $\boldsymbol{P(B) = \frac{28}{66} = \frac{14}{33}}$.

Next, S2 selected and determined the strategies for calculating the probability of choosing one blue marble and one white marble. Figure 12 shows the problem-solving strategies.

In Figure 12, S2 selected and determined three strategies for solving the problem. S2 wrote $\boldsymbol{n(B) = C_1^8, n(P) = C_1^4, and\ P(B \cap P) = \frac{n(B \cap P)}{n(S)}}$. However, before writing the strategies, S2 assumed B as the probability of selecting

Figure 11. *S2's Strategy for Calculating the Probability of Selecting Two Blue Marbles*

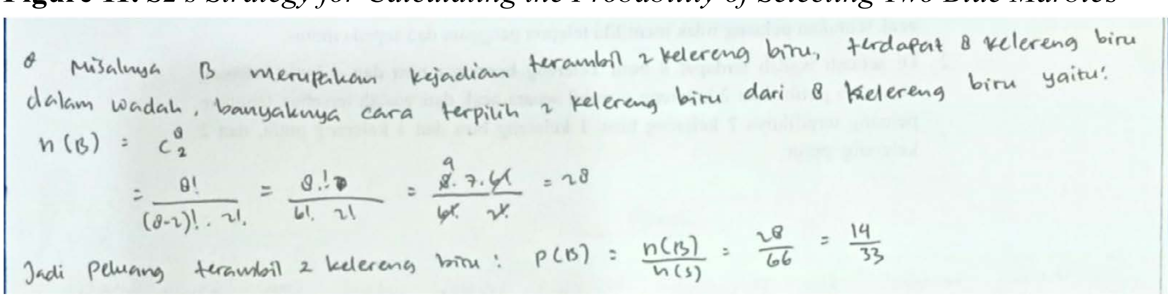

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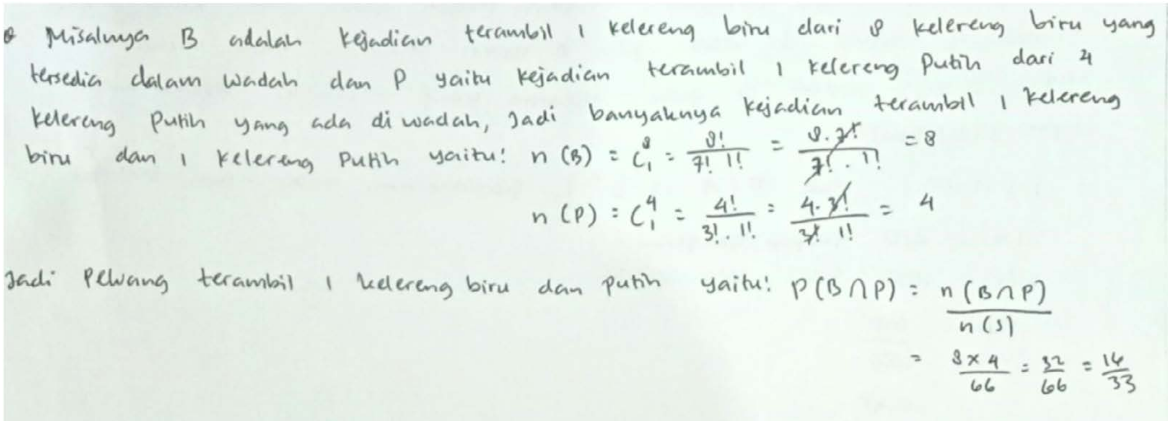
Figure 12. *S2's Strategies for Calculating the Probability of Choosing Blue and White Marbles*


Misalnya B adalah kejadian terambil 1 kelereng biru dari 8 kelereng biru yang tersedia dalam wadah dan P yaitu kejadian terambil 1 kelereng putih dari 4 kelereng putih yang ada di wadah, Jadi banyaknya kejadian terambil 1 kelereng biru dan 1 kelereng putih yaitu: $n(B) = C^8_1 = \frac{8!}{7!\,1!} = \frac{8.7!}{7!.1!} = 8$

$n(P) = C^4_1 = \frac{4!}{3!.1!} = \frac{4.3!}{3!\,1!} = 4$

Jadi peluang terambil 1 kelereng biru dan putih yaitu: $P(B \cap P) = \frac{n(B \cap P)}{n(S)}$

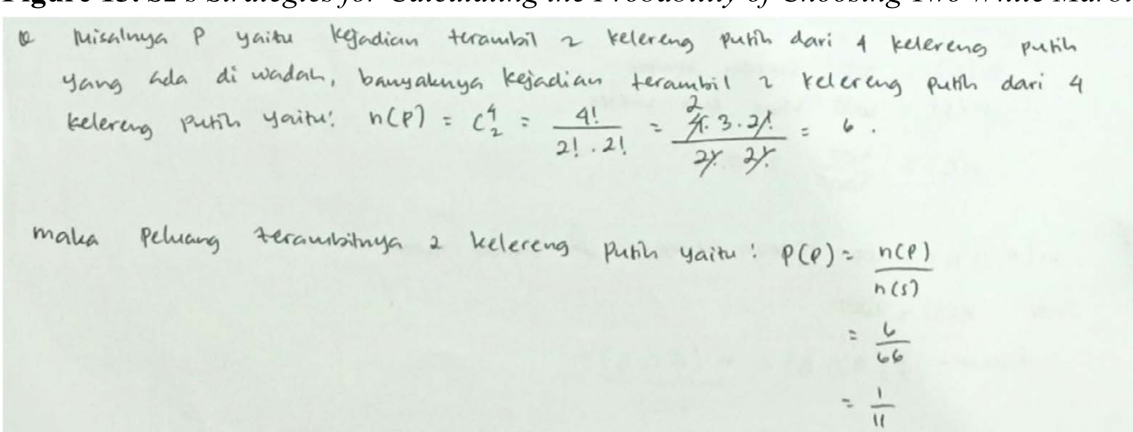
$= \frac{8 \times 4}{66} = \frac{32}{66} = \frac{16}{33}$

one blue marble out of eight marbles available, and P as the probability of choosing one white marble out of four white marbles available. Then, S2 applied the first and second strategies by involving the following mathematical calculations: $\boldsymbol{n(B) = \frac{8!}{7!1!} = \frac{8.7!}{7!.1!} = 8\ combinations}$, $\boldsymbol{n(P) = \frac{4!}{3!1!} = \frac{4.3!}{3!.1!} = 4\ combinations}$. After that, S2 discovered the answer to the problem and applied the third strategy to draw a conclusion by involving the following mathematical calculation: $\boldsymbol{P(B \cap P) = \frac{8\times4}{66} = \frac{32}{66} = \frac{16}{33}}$.

At the final stage, S2 selected and determined two strategies to calculate the probability of choosing two white marbles (Figure 13).

In Figure 13, S2 selected and determined two strategies for calculating the probability of choosing two white marbles. As a result, S2 wrote the following formulas $\boldsymbol{n(P) = C^4_2\ and\ P(P) = \frac{n(P)}{n(S)}}$. Before using the formulas, S2 assumed that P was the probability of choosing two white marbles out of four white marbles available. Then, S2 applied the first strategy by involving a mathematical

Figure 13. *S2's Strategies for Calculating the Probability of Choosing Two White Marbles*


Misalnya P yaitu kejadian terambil 2 kelereng putih dari 4 kelereng putih yang ada di wadah, banyaknya kejadian terambil 2 kelereng putih dari 4 kelereng putih yaitu: $n(P) = C^4_2 = \frac{4!}{2!.2!} = \frac{4.3.2!}{2!\,2!} = 6$.

maka peluang terambilnya 2 kelereng putih yaitu: $P(P) = \frac{n(P)}{n(S)}$

$= \frac{6}{66}$

$= \frac{1}{11}$

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calculation, which he wrote in detail as $n(P) = \frac{4!}{2!2!} = \frac{4.3.2!}{2!.2!} = 6\ combinations$. Next, S2 discovered the answered and drew a conclusion on the problem by implementing the second strategy, where he used the mathematical calculation as follows: $P(P) = \frac{6}{66} = \frac{1}{11}$.

These problem-solving processes were reexamined and confirmed through an interview; here is an excerpt of it:

R: How did you solve question number 2?
S2: Using the formulas for combinations and probability
R: Is there another way that can be used to solve question number 2?
S2: Yes, there is, by using the formula for combinations.
R: What are the steps to solve a problem like this?
S2: Read the question carefully and understand it, then determine the sample size and calculate the probability of the event.

Based on this excerpt, S2 employed two strategies to address the problem at hand: utilizing the formula for combinations and leveraging the concept of probability. In addition to employing the probability formula, S2 used the combination formula. Additionally, S2 provided a comprehensive explanation of the sequential process involved in resolving similar problems.

We conducted a more in-depth interview to examine the conclusion made by S2 on the probability of a compound event.

Table 2. *The Problem-Solving Processes Undergone by S1 and S2 when Completing Reasoning Tasks*

Subject	Indicator	Description	Type of Reasoning
S1	Understanding the Problem	Able to write the information known from the task	Imitative reasoning
		Able to determine what the task asks	
	Planning the Solution	Able to select and determine the appropriate strategy for solving the problem	
		The implemented strategy has been used before	
	Implementing the Problem-Solving Strategy(s)	Able to implement the problem-solving steps appropriately	
	Drawing a conclusion	Able to draw the correct conclusion	
		Conduct a reexamination of the result	
S2	Understanding the problem	Able to write the information known from the task	Imitative reasoning
		Able to determine what the task asks	
	Planning the Solution	Able to select and determine the appropriate strategy for solving the problem	
		The implemented strategy has been used before	
	Implementing the Problem-Solving Strategy(s)	Able to implement the problem-solving steps appropriately	
	Drawing a conclusion	Able to draw the correct conclusion	
		Conduct a reexamination of the result	

Note: derived from research.

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R: Could you write the conclusion on the problem using your own words?
S2: Yes, I could
R: Are you sure about your answer?
S2: Yes, I'm sure about it
R: Have you examined the result?
S2: Yes

The interview excerpt suggests that S2 demonstrated the ability to express his thoughts effectively through written responses. Moreover, S2 exhibited self-assurance in the correctness of his responses and conscientiously double-checked his work before presenting it to the teacher. The problem solving process that S1 and S2 undertook when completing the reasoning task is shown as follows:

Discussion

The current study examined the mathematical reasoning of students with Kolb's converging learning style. Convergers typically possess strong problem-solving and decision-making skills. The data analysis in this study demonstrates that the convergers exhibited proficiency in executing the four stages of the reasoning process.

During the "understanding the problem" stage, the subjects referred to the acquired information. During the "planning the solution" stage, the subjects selected and established the most suitable strategy for effectively resolving the problem. The subjects employed imitative reasoning by selecting a previously studied strategy that had been successfully used for a similar problem. At the "implementing the problem-solving strategy" stage, the subjects could apply the problem-solving strategy by following the appropriate steps. The subjects utilized imitative, memory-based, and algorithmic reasoning to complete all reasoning tasks. The research subjects utilized memory to recall key words when constructing a mathematical model.

Algorithms were commonly employed in problem-solving tasks as the research subjects prioritized the memorization of procedures and the accurate application of algorithms to the given questions. During the final stage, referred to as "drawing a conclusion," the subjects successfully derived accurate conclusions and identified solutions to the problem strategies that had been correctly resolved. The subjects also verified if their answers aligned with the requirements of the tasks. However, S-2 made an error in question 2. Based on the problem-solving outcomes, both subjects were classified as individuals with imitative reasoning, as they employed reasoning derived from prior experiences to solve problems.

The converging learning style combines Abstract Conceptualization (AC) and Active Experimentation (AE), integrating both cognitive processes and practical application. Convergers demonstrate strong problem-solving skills, employ a systematic approach to their work, and are adept at drawing accurate conclusions. Students with a converging learning style exhibit strong problem-solving and decision-making skills (Gasteiger *et al.*, 2020; K. H. Lee, 2017; Singer *et al.*, 2017). These students also excel in applying ideas and theories to practical situations.

The implication of this research is that mathematics teachers must identify their students' mathematical reasoning in solving probability problems. The results of this research showed that students with a converger learning style were able to solve problems at each stage of reasoning (Hirschfeld-Cotton, 2008; Khozaei et al., 2022; Pitta-Pantazi &

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Christou, 2009). Students with a converger learning style could solve problems better than students with other learning styles. This finding is corroborated by those of K. H. Lee (2017) and Singer et al. (2017), who stated that students with a converger learning style can perform better in mathematics than students with other learning styles. It is hoped that the results of this research will be useful for teachers in carrying out the learning process by considering students' learning styles and mathematical reasoning in problem-solving. Teachers can train students' mathematical reasoning by focusing on their learning styles. This strategy may lead to students' increased performance in each stage of reasoning when solving probability problems.

Conclusions

The study found that students with converging learning styles predominantly used imitative reasoning when solving probability problems. The research subjects demonstrated a tendency to identify the known information accurately and comprehensively from the tasks, as well as to respond accurately to specific problems. While they possessed an understanding of probability, they struggled with the ability to reason about arithmetic sequences. They exhibited the ability to use appropriate problem-solving approaches, albeit occasionally lacking precision in articulating them in writing. The subjects also demonstrated proficiency in utilizing the concept of probability, implementing strategies to address probability-related problems, and articulating the sequential steps involved in problem-solving. Furthermore, the participants exhibited the ability to employ problem-solving strategies and derive conclusions accurately. These individuals exhibited a tendency to place trust in their answer and subsequently verified its accuracy.

The current study also revealed a relationship between imitative reasoning and students' prior problem-solving experience. In this scenario, students solely replicate preexisting solutions. Therefore, mathematics teachers are expected to familiarize students with open-ended questions that promote creative reasoning rather than merely emphasizing memorization. Mathematics teachers should possess the ability to offer diverse learning experiences by understanding the learning styles of their students. This ensures that all students have equal opportunities to comprehend the instructional content.

Other researchers may engage in experimental research aimed at enhancing students' mathematical reasoning skills in problem-solving. Future researchers can also conduct development research to create a product, such as a mathematics module or tabloid, that can facilitate problem-solving learning for students with different learning styles. Further research can be conducted by creating a Kolb learning style questionnaire or a mathematical reasoning test.

Conflict of Interest

The authors declare no competing interests with the research, authorship, and/or publication of this article

Author contribution statement

D.R.A. conceived the idea of the research presented. M.Ik. collected the data. The four authors (D.R.A., S.R, S.Z.T, and M.Ik.) actively participated in the development of the theory, methodology, data organisation and analysis, discussion of

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results, and approval of the final version of the work. All the authors declare that the final version of this paper was read and approved. The total contribution percentage for the conceptualization, preparation, and correction of this paper was as follows: D.R.A. 40%, S.R 20%, S.Z.T 20%, and M.Ik 20%

Data availability statement

The data supporting the results of this study will be made available by the corresponding author (D.R.A.) upon reasonable request.

Preprint

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